

# Using a Mach-Zehnder interferometer to measure the phase retardations of wave plates

Fang-Wen Sheu and Shu-Yen Liu

Department of Applied Physics, National Chiayi University, Chiayi 60004, Taiwan

Tel: +886-5-2717993; Fax: +886-5-2717909; E-mail: fwsheu@mail.ncyu.edu.tw

**Abstract:** A wave plate is a commonly used optical element in optical experiments. In this report, we have achieved measuring the phase retardations of a half-wave plate and a quarter-wave plate using a Mach-Zehnder interferometer. Besides, when we rotate the half-wave plate's c-axis from the vertical to the horizontal directions, or vice versa, the phase retardations of the two orthogonally polarized beams are observed to be exchanged between  $180^\circ$  and  $-180^\circ$ . In addition, we also predict the expected results by the Jones calculus theory in order to check the experimental results. This system can be applied to explore intuitively the birefringence characteristics of anisotropic materials in an optics teaching laboratory.

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**OCIS codes:** (120.3180) Interferometry; (120.5050) Phase measurement; (260.1440) Birefringence

## 1. Introduction

A wave plate is a commonly used optical element in optical experiments. The phase retardation of a wave plate is an important value that influences the measurement results significantly. In this report, an interferometric method is presented to measure the phase retardation of a wave plate intuitively. We achieve measuring the phase retardations of a half-wave plate and a quarter-wave plate using a Mach-Zehnder interferometer. When we rotate the half-wave plate's c-axis from the vertical to the horizontal directions, or vice versa, the phase retardations of the two orthogonally polarized beams are observed to be exchanged between  $180^\circ$  and  $-180^\circ$ . In addition, we also evaluate the results in theory by Jones calculus in order to check the experimental results.

## 2. Experimental Setup and Principle of Measurement

The schematic diagram and photograph of the experimental setup are shown in Fig. 1. The output light of a He-Ne laser is linearly polarized with an angle of  $45^\circ$  with respect to the horizontal direction, and is of wavelength at 632.8 nm. The laser beam passes through a beam splitter and is divided into two parts. The reflected light passes through the free space and the transmitted light passes through a test wave plate with c-axis in the x or y direction, and a mirror mounted on a piezoelectric transducer (PZT) which is driven by a triangle-wave alternating voltage of low frequency. In this way the phase of the transmitted light could be modulated linearly. Both of the reflected light and the transmitted light are incident on the second beam splitter and interfere with each other. Then the interfering light is divided into two orthogonally polarized parts by a polarizing beam splitter. The plane-polarized transmitted light passes through an objective lens and arrives at a photodetector. The surface-polarized reflected light passes through another objective lens and then is measured by another photodetector. The two sinusoidally oscillating optical intensities are recorded by an oscilloscope, which reveals the phase retardation produced by the test wave plate.

The theoretical analysis of the principle of measurement is derived as follows. At first, the He-Ne laser beam passes through a beam splitter and is divided into two parts. The electric field of the transmitted beam passing through the test wave plate and the mirror mounted on a PZT is expressed by

$$\vec{E}_w = A'_x \exp[j\beta_x L + j\beta_0(z_0 - L + a \times t)] \hat{x} + A'_y \exp[j\beta_y L + j\beta_0(z_0 - L + a \times t)] \hat{y} \quad (1)$$

The  $\beta_x$  and  $\beta_y$  are the propagation constants of the two orthogonally polarized modes in the principal axes of the wave plate. The  $\beta_0$  is the propagation constant of light in free space. The  $\hat{x}$  and  $\hat{y}$  are the unit vectors

along the two birefringent principal axes. The  $z_0$  is the total distance of the optical path. The  $L$  is the thickness of the test wave plate. The coefficient  $a$  is the varying rate of the optical path induced by the linear displacement of the vibrating PZT. The electric field of another beam passing through the free-space optical path is

$$\vec{E}_r = A_x \exp[j\beta_0 \times z_0] \hat{x} + A_y \exp[j\beta_0 \times z_0] \hat{y} \quad (2)$$

Finally, the two optical beams recombine in the second beam splitter and interfere with each other component at the reflection or the transmission ports. One of the two interfering beams will be divided into the plane-polarized and the surface-polarized beams by a polarizing beam splitter, with optical intensities

$$\begin{aligned} I_x(t) &= \left| A_x \exp[j\beta_0 \times z_0] + A'_x \exp[j\beta_x L + j\beta_0(z_0 - L + a \times t)] \right|^2 \\ &= |A_x|^2 + |A'_x|^2 + 2A_x A'_x \times \cos[\beta_0 \times z_0 - \beta_x L - \beta_0(z_0 - L + a \times t)] \\ &= |A_x|^2 + |A'_x|^2 + 2A_x A'_x \times \cos[(\beta_0 - \beta_x)L - \beta_0 \times a \times t] \end{aligned} \quad (3)$$

$$\begin{aligned} I_y(t) &= \left| A_y \exp[j\beta_0 \times z_0] + A'_y \exp[j\beta_y L + j\beta_0(z_0 - L + a \times t)] \right|^2 \\ &= |A_y|^2 + |A'_y|^2 + 2A_y A'_y \times \cos[\beta_0 \times z_0 - \beta_y L - \beta_0(z_0 - L + a \times t)] \\ &= |A_y|^2 + |A'_y|^2 + 2A_y A'_y \times \cos[(\beta_0 - \beta_y)L - \beta_0 \times a \times t] \end{aligned} \quad (4)$$

Thus, the phase delay between the two sinusoidally oscillating optical intensities  $I_x(t)$  and  $I_y(t)$ , which is induced by the optical path vibration due to the PZT driving, is given by

$$\phi = (\beta_0 - \beta_x)L - (\beta_0 - \beta_y)L = (\beta_y - \beta_x)L \quad (5)$$

As a consequence, the phase delay  $\phi$  between  $I_x(t)$  and  $I_y(t)$  happens to equal the phase retardation of the two orthogonally polarized fields produced by the birefringent wave plate, which is to be determined. As long as the magnitude of the PZT driving voltage is large enough to make  $I_x(t)$  and  $I_y(t)$  oscillate by a phase change more than  $2\pi$  in the time of half a period of the PZT alternating signals, we can easily measure the phase delay between the two sinusoidally oscillating scan traces on the oscilloscope, which represent the signals from the two photodetectors.

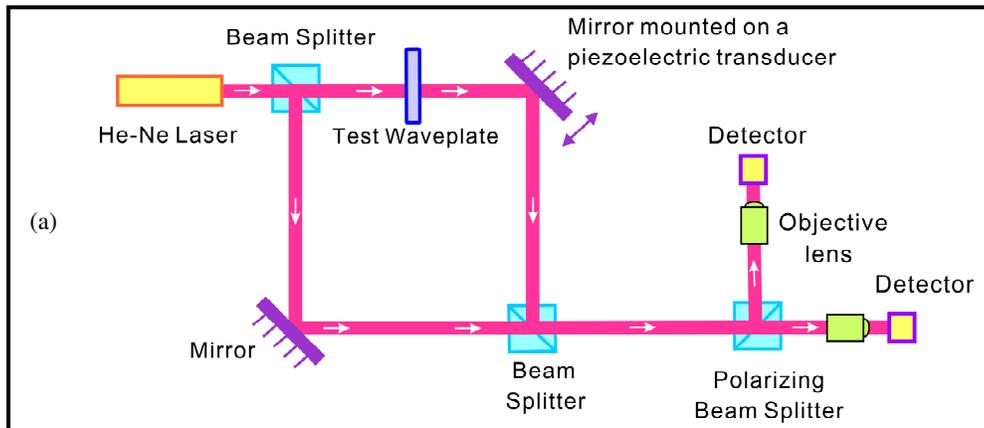




Fig. 1. The (a) schematic diagram and (b) photograph of the experimental setup.

### 3. Measurement of the phase retardations of various wave plates

Figure 2 shows the measured results of the two sinusoidally oscillating optical intensities on the oscilloscope. The phase retardation of the two orthogonally polarized interfering beams is  $0^\circ$ , when there is no any wave plate placed in the set up. The phase retardation becomes  $90^\circ$  when a quarter-wave plate is inserted, and the phase retardation becomes  $180^\circ$  when a half-wave plate is tested. Hence we have achieved measuring the phase retardations of wave plates using a Mach-Zehnder interferometer in a more intuitive way.

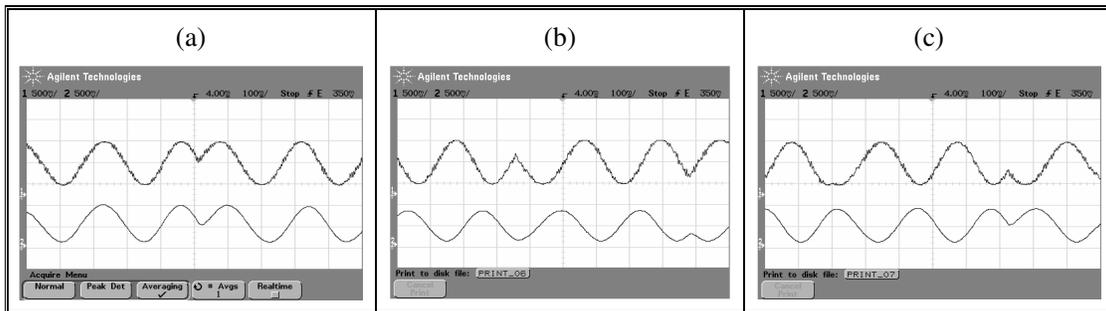


Fig. 2. The oscilloscope scan traces for the two orthogonally polarized interfering beams. (a) Without any wave plates, the phase retardation is  $0^\circ$ . (b) With a quarter-wave plate, the phase retardation is  $90^\circ$ . (c) With a half-wave plate, the phase retardation is  $180^\circ$ .

### 4. Measurement of the phase retardations of a rotated half-wave plate

Figure 3 shows the theoretical and experimental results of the phase retardation of the two orthogonally polarized beams when we rotate the half-wave plate by an angle of  $22.5^\circ$  successively. We can see that the phase retardations of the two orthogonally polarized beams are exchanged between  $180^\circ$  and  $-180^\circ$  when we rotate the half-wave plate's c-axis from the vertical to the horizontal directions, or vice versa. The experimental results agree quite well with the theoretical analysis performed by the MATHEMATICA software, as shown in the Appendixes.

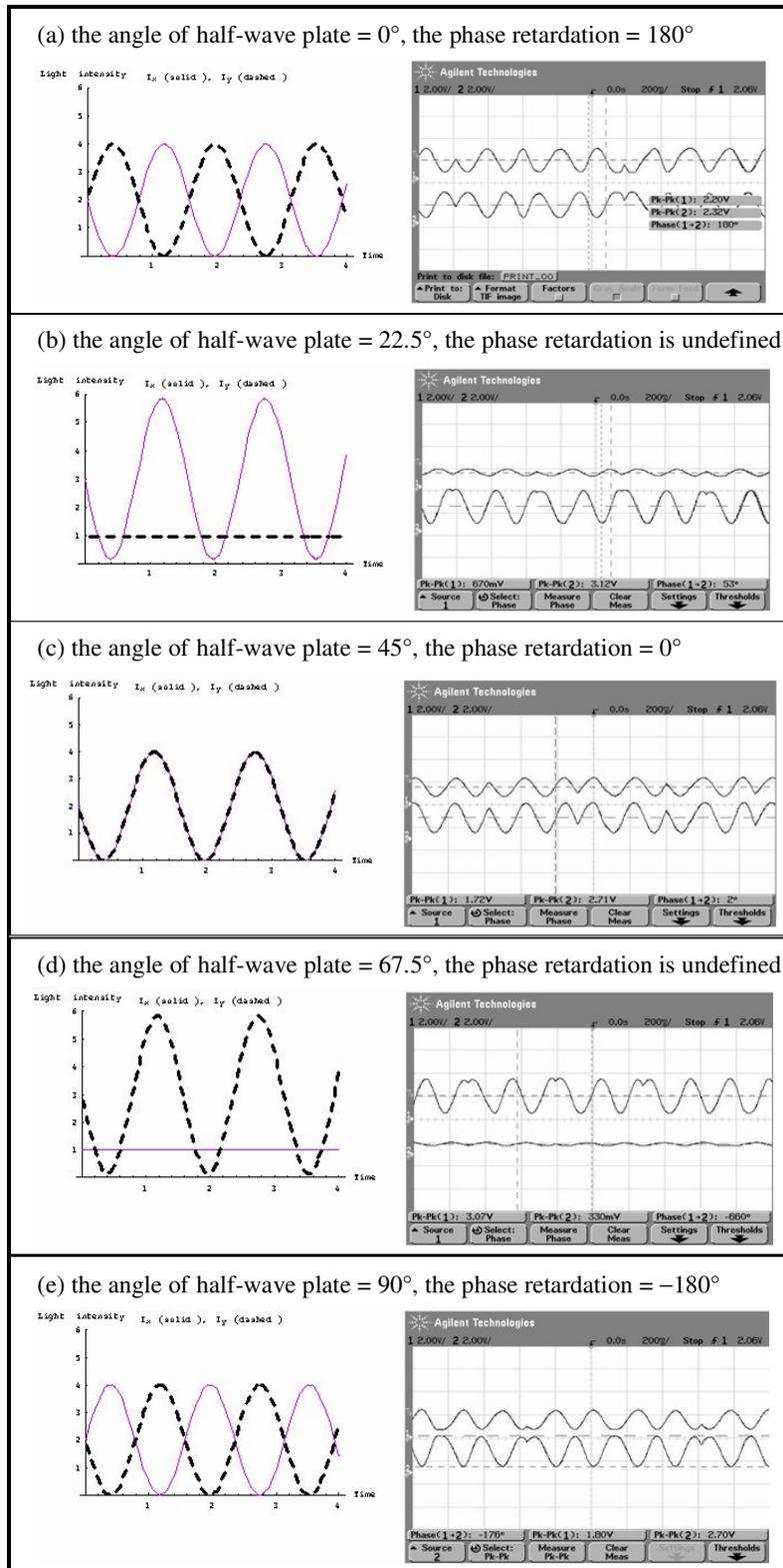


Fig. 3. The theoretical and experimental results for the two orthogonally polarized interfering beams, when we rotate the half-wave plate by (a)  $0^\circ$ , (b)  $22.5^\circ$ , (c)  $45^\circ$ , (d)  $67.5^\circ$ , and (e)  $90^\circ$ . In the simulation parts, the surface-polarized light signal is shown as the dashed line and the plane-polarized light signal is shown as the solid line.

## 5. Conclusion

The experimental results and theoretical analysis show good agreement and prove that using a Mach-Zehnder interferometer is a good technique to measure the phase retardations of wave plates easily and quickly. This system can be applied in an optics teaching laboratory to explore the birefringence characteristics of anisotropic materials in a more intuitive manner.

## Acknowledgements

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## References

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## Appendixes

The MATHEMATICA code of the theoretical analysis of the phase retardations of a rotated half-wave plate:

```
(* -----Half Waveplate----- *)
Γ = π;
For[ψ = 0, ψ ≤ π / 2, ψ = ψ + π / 8,
  W = (Cos[ψ] -Sin[ψ]) . (Exp[-i n Γ / 2] 0) . (Cos[ψ] Sin[ψ]);
      (Sin[ψ] Cos[ψ]) . (0 Exp[i n Γ / 2]) . (-Sin[ψ] Cos[ψ]);
  β0 = 2; a = 10; b = 2; zW = a + b × t; z0 = 10;
  EW = (Exp[-i β0 × zW] 0) . W . (1);
      (0 Exp[-i β0 × zW]) . W . (1);
  Ef = (Exp[-i β0 × z0] 0) . (1);
      (0 Exp[-i β0 × z0]) . (1);
  ETotal = EW + Ef;
  Ex = (1 0) . ETotal; Ey = (0 1) . ETotal;
  Ix = Norm[Ex]2; Iy = Norm[Ey]2;
  T = 2 π / (β0 × b);
  Pdcx = N[∫0T Ix dt] / T; Print["Idcx=", Pdcx];
  Qdcy = N[∫0T Iy dt] / T; Print["Idcy=", Qdcy];
  xsol = FindRoot[Ix == Pdcx, {t, 1}]; Print["xsol=", xsol];
  ysol = FindRoot[Iy == Qdcy, {t, 1}]; Print["ysol=", ysol];
  phase = 360 × ((t / xsol) - (t / ysol)) / T; Print["phase=", N[phase]];
  Print["Waveplate: angle ψ = ", ψ, " , phase retardation Γ = ", Γ];
  Plot[{Ix, Iy}, {t, 0, 4}, PlotRange → {0, 6}, AxesLabel → {"Time", "Light intensity"},
    PlotLabel → "Ix (solid), Iy (dashed)",
    PlotStyle → {{Hue[0.8]}, {Thickness[0.015], Dashing[{0.03, 0.03]}}];
  Print["-----"]
];
(* -----Half Waveplate----- *)
```