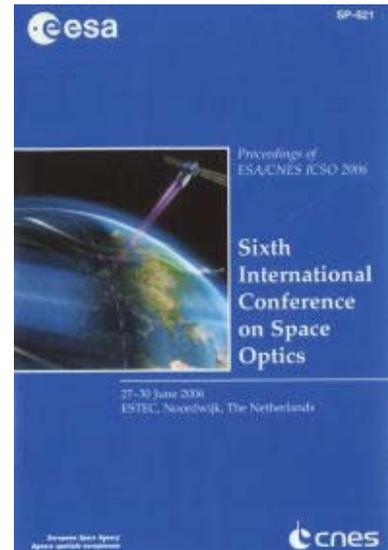


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Three-dimensional electric field analysis in a multi-axial beam combiner for nulling interferometry

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THREE-DIMENSIONAL ELECTRIC FIELD ANALYSIS IN A MULTI-AXIAL BEAM COMBINER FOR NULLING INTERFEROMETRY

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ABSTRACT

We perform a calculation of the vectorial field distribution in the focal plane of a multi-axial beam combiner and show the fundamental limitations with respect to the longitudinal component of the polarization of such a combiner for nulling interferometry.

Key words: Interferometry, Nulling interferometry, Polarization.

1. INTRODUCTION

Direct detection of Earth-like exoplanets is very challenging because of the huge brightness contrast between the star and the planet (10^6 at $10 \mu\text{m}$) and their small angular separation (typically 0.1 arcsec). One promising technique that should allow such a detection is nulling interferometry (Bracewell, 1978), which consists in looking at a star-planet system with an array of telescopes, and then combining the light from these telescopes in order to have destructive interference for the star light and simultaneously (partially) constructive interference for the planet light. The ratio between the intensities corresponding to constructive and destructive interference is called the rejection ratio. To be able to detect an Earth-like planet, this ratio should be of the order of 10^6 .

In order to create an interference, we must combine the light coming from the different telescopes. With conventional optics, there are two types of beam combination: uni-axial (or Michelson-type) and multi-axial (or Fizeau-type) combination (see Figure 1). In a uni-axial combiner, beams are superimposed with beam-splitters to form only one beam. In a multi-axial combiner, different beams are imaged with a focusing optics and overlap only in the

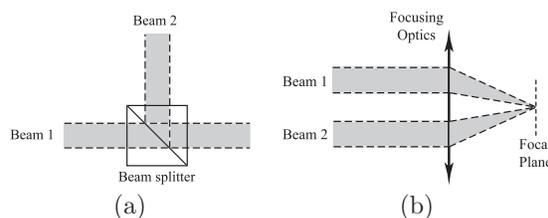


Figure 1. (a) Uni-axial and (b) Multi-axial combination.

image plane. The advantage of the second method is that it can only involve mirrors and can therefore be easily achromatic. Unfortunately, the focusing optics will introduce a longitudinal component of the electric field, that will limit the performance of a multi-axial nulling interferometer.

In this paper, we will perform a rigorous three-dimensional electric field analysis in the focal plane of a focusing optics in order to quantify the limitations of a multi-axial beam combiner with respect to longitudinal polarization. In Section 2, we describe the theory that we used to calculate the three-dimensional electric field in the focal plane. In Section 3, we present the results of these calculations in the case of a two- and a three-beam multi-axial beam combiner. Our conclusions are then summarized in Section 4.

2. THEORY

In a multi-axial beam combiner, the focusing optics will rotate the wave vectors (k_1 and k_2) and therefore the vibration planes of the different beams, in such a way that, depending on the initial polarization, a longitudinal component will be created in the focal plane (see Figure 2). Suppose the beams are

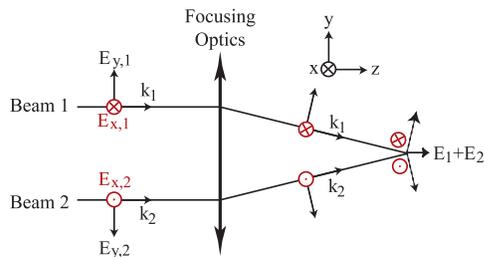


Figure 2. Schematic combination of linearly polarized beams. Depending on the initial orientation of the polarization, the focusing optics introduces a longitudinal component of the electric field.

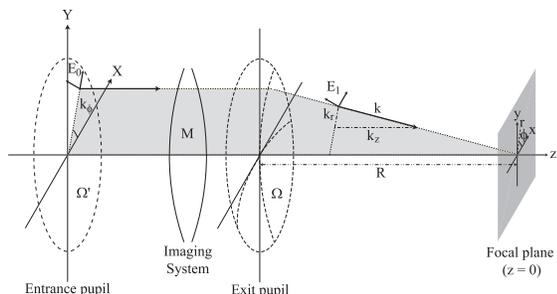


Figure 3. Schematic overview of the studied configuration. Light distribution in the entrance pupil $E_0(k_r, k_\phi, 0)$ is mapped to the exit pupil $E_1(k_r, k_\phi, k_z)$ via an aplanatic imaging system, denoted by the operation M . The field distribution in the focal region is then obtained by integration over the exit pupil. The focal plane is the plane $z = 0$.

linearly-polarized in the y -direction, we can see on Figure 2 that the focusing optics creates in the focal plane a z -component of the polarization that will limit the rejection ratio. The larger the numerical aperture, the larger that z -component. If the beams are linearly-polarized in the x -direction, the electric field remains transversal and the rejection ratio is theoretically infinite. Note that for both cases, we considered equal amplitudes and a π -phase shift between the two beams since we want on-axis destructive interference.

With this simple approach, we can show that the longitudinal component of the electric field will limit the rejection ratio of a multi-axial nulling interferometer. In order to be more rigorous, we performed an analysis of the three-dimensional electric field in the focal plane of the focusing optics.

Let us consider the aplanatic imaging system depicted in Figure 3. This imaging system maps the electric field distribution in the entrance pupil $E_0(k_r, k_\phi, 0)$ to the exit pupil $E_1(k_r, k_\phi, k_z)$. The exit pupil is a spherical shell with radius R . To describe the electric field, we introduce a set of cylindrical coordinates in the exit pupil $\mathbf{k} = (k_r, k_\phi, k_z)$

and in the focal region $\mathbf{r} = (r, \phi, z)$, where the plane $z = 0$ is the focal plane. If we consider a monochromatic time-harmonic electric field, we can calculate the electric field in the focal plane using diffraction integrals described by Richards & Wolf (1959) and van de Nes (2004). These diffraction integrals are valid in the Debye approximation, therefore our point of observation should not be too close to the spherical shell Ω over which the integration takes places. Nevertheless, it is more convenient to integrate over the entrance pupil Ω' rather than over the exit pupil Ω . Since we consider an aplanatic imaging system, the transition from the entrance pupil to the exit pupil can be considered as a rotation of the wave vector, described by the propagation matrix \mathbf{M}

$$M = \frac{1}{k} \begin{pmatrix} k_z \cos^2 k_\phi + k \sin^2 k_\phi & (k_z - k) \cos k_\phi \sin k_\phi \\ (k_z - k) \cos k_\phi \sin k_\phi & k_z \sin^2 k_\phi + k \cos^2 k_\phi \\ -k_r \cos k_\phi & -k_r \sin k_\phi \end{pmatrix}. \quad (1)$$

Taking these considerations into account, the electric field in the focal plane is given (van de Nes, 2004) by

$$\mathbf{E}(r, \phi, 0) = -\frac{iR}{2\pi} \iint_{\Omega'} \sqrt{\frac{k_z}{k}} \frac{\mathbf{M} \cdot \mathbf{E}_0(k_r, k_\phi)}{k_z} \exp[irk_r \cos(k_\phi - \phi)] k_r dk_r dk_\phi, \quad (2)$$

where $k = 2\pi/\lambda$ is the wave number. This expression allows us to calculate the three-dimensional electric field in the focal plane $\mathbf{E}(r, \phi, 0)$ given a certain field distribution $\mathbf{E}_0(k_r, k_\phi)$ in the entrance pupil. The intensity distribution in the focal plane is then given by

$$I(r, \phi) = |E_x(r, \phi)|^2 + |E_y(r, \phi)|^2 + |E_z(r, \phi)|^2. \quad (3)$$

The rejection ratio is then defined by

$$R = \frac{\max[I(r, \phi)]}{I(r=0, \phi)}. \quad (4)$$

3. SIMULATIONS

On-axis destructive interference required in nulling interferometry can be performed with N beams. Nevertheless, in practice, we often use a small number of beams. Therefore, we will calculate the three components of the electric field in the case of two- and three-beam multi-axial combiners. We will also distinguish on- and off-axis beam combiners.

In all simulations, we consider linearly-polarized beams with a diameter $D = 2$ cm and a wavelength of 600 nm. We choose a focusing optics with a focal length of 60 cm and the baseline, i.e. the distance between the beams before combination, is $L = 2.6$ cm.

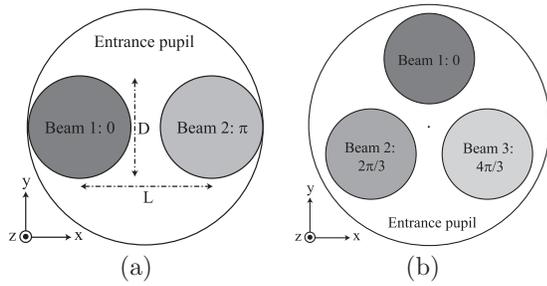


Figure 4. Schematic entrance pupil for the combination of (a) two beams located along the x -axis and (b) three beams. There is a phase-shift between the beams in order to have on-axis destructive interference. The diameter of the beams is D and the distance between the beams in the entrance pupil is L .

In this section, we will first consider the on-axis multi-axial combination of two beams located along the x -axis (see Figure 4(a)). The three normalized components of the electric field in the case of linearly-polarized beams along the x - and the y -axis are respectively depicted in Figure 5(a) and 5(b).

As we expected from the simple approach depicted in Figure 2, the rejection ratio corresponding to linearly-polarized beams along the y -axis is infinite since the three on-axis ($x = y = 0$) components of the electric field are equal to 0. In the case of linear polarization along the x -axis, a large on-axis longitudinal (E_z) component limits the rejection ratio to $R = 1500$.

Note that we would find comparable results if the two beams were off-axis in the x -direction. In the case of x -polarized beams, the rejection ratio would still be of the order of $R = 1500$, while in the case of y -polarized beams, the rejection ratio would now be limited to $R = 2 \times 10^5$ (instead of infinity for on-axis combination). Indeed, the on-axis longitudinal component would not exactly be equal to zero because of asymmetry.

The rejection ratio also depends on the numerical aperture of the focusing optics. We can define an effective numerical aperture NA_{eff} as the ratio between the semi-baseline $L/2$ and the focal length of the focusing optics f ,

$$NA_{\text{eff}} = \frac{L}{2f}. \quad (5)$$

The rejection ratio as a function of the effective numerical aperture is depicted in Figure 6, for different values of the ratio L/D . The minimal value of this ratio is 1, corresponding to the case where the beams touch each other. We can see that the rejection ratio is inversely proportional to the square of the effective numerical aperture. The ratio L/D does not drastically affect the rejection ratio.

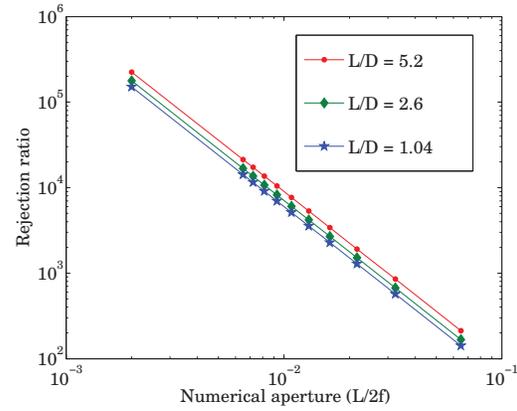


Figure 6. Rejection ratio as a function of the effective numerical aperture $L/2f$ for different values of the ratio L/D .

In a real nulling interferometer, there is most of the time a single-mode optical fiber after combination for modal filtering. To optimize the coupling efficiency on the fiber, the beams should be as close as possible ($L/D = 1$) and the focal length is chosen to match the numerical aperture of the fiber (typically 0.12). These considerations give an effective numerical aperture $NA_{\text{eff}} = 0.06$ and a typical rejection ratio of the order of 10^2 , which is definitely too low for Earth-like exoplanet detection.

In this section, we will consider the multi-axial combination of three beams, as depicted in Figure 4(b). In this case, for both x - and y -polarization and for both on- and off-axis combination, the z -component of the polarization limits the rejection ratio to $R = 1700$. Indeed, the rejection ratio should be comparable for both polarization since there is no geometrically-preferred direction, as it is the case for the two-beam combiner. Note that if the three beams were arranged in a linear configuration, an infinite rejection ratio would be possible for polarization direction perpendicular to the baseline.

4. OUTLOOK AND CONCLUSIONS

We performed a three-dimensional electric field analysis in the focal plane of a multi-axial nulling interferometer. In the case of a two-beam combiner, we have seen that the rejection ratio was much higher when the polarization is perpendicular to the baseline. If the beams are linearly-polarized along the baseline direction, the rejection ratio would be drastically limited. We also have seen that the rejection ratio is inversely proportional to the square of the effective numerical aperture. In the case of a three-beam combiner, the rejection ratio is always limited (except in the linear-configuration case). In any case, there is always one component of the polarization for

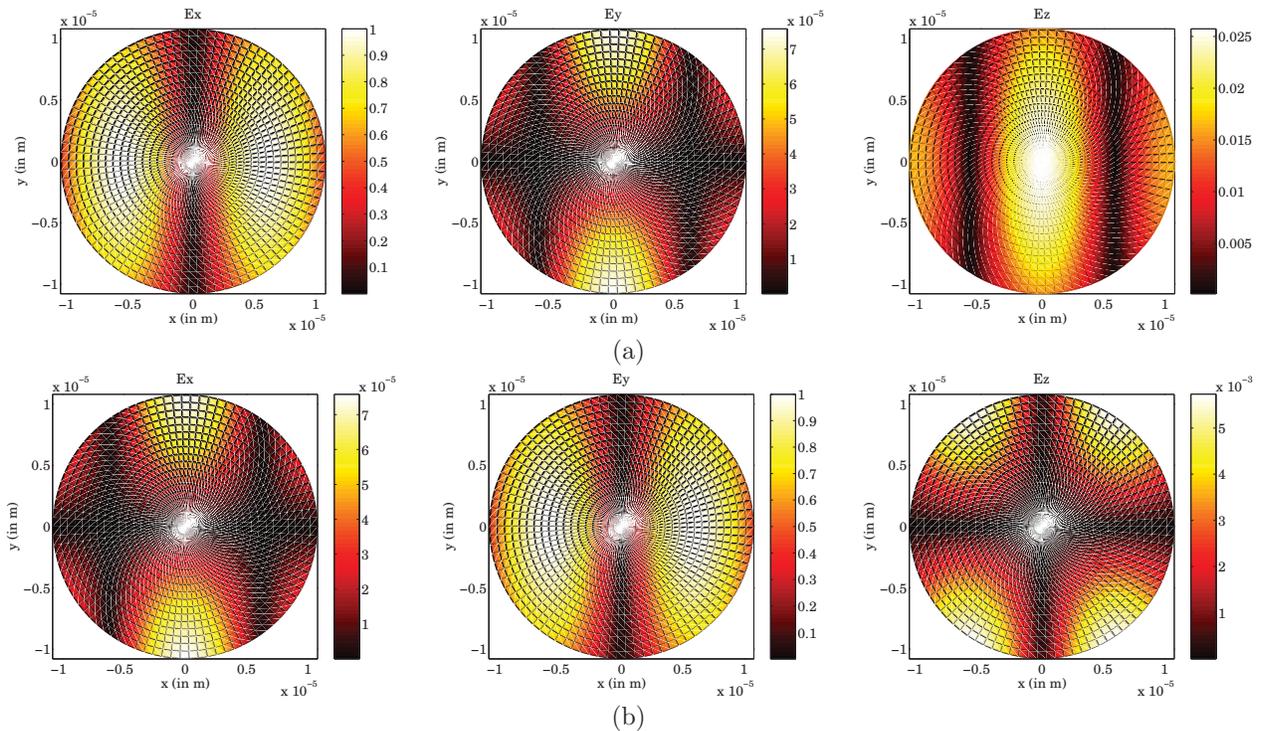


Figure 5. Three components of the electric field (E_x, E_y and E_z) in the case of a two-beam multi-axial combiner. The baseline is along the x -axis and the beams are linearly-polarized along the (a) x -axis and (b) y -axis.

which the rejection ratio is limited, preventing us to work in a dual-polarization mode.

With this study, we conclude that the rejection in a multi-axial nulling interferometer would be limited due to the longitudinal component of the electric field. Therefore, direct detection of an Earth-like exoplanet would not be possible as such. However, on every nulling interferometer, wavefront filtering is needed either with a pinhole or a single-mode optical fiber. In this case, the transition between air and medium will change the spatial distribution of the three field components and their relative strength. A quantitative analysis of this issue is not trivial but will be necessary to fully understand the problem.

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REFERENCES

- Bracewell R., 1978, Nature 274, 780
- Richards B., Wolf E., 1959, Proc. Roy. Soc. A 253, 358
- van de Nes A.S., Billy L., Pereira S.F., Braat J.J.M., 2004, Opt. Exp. 12, 1281