The Art of Insight in Optical Science and Engineering

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Perhaps the most common reason undergraduate students decide to major in science and engineering is that they like to understand how "things" work. In this section, I give a brief overview of an instructional methodology—the art of insight—which brings a more conceptual and intuitive approach to the educational process to directly address this interest.

To illustrate, I use the example of the wave equation in physical optics for the propagation of the electric field amplitude E_y:

$$\frac{\partial^2 E_y}{\partial t^2} = v_\phi^2 \frac{\partial^2 E_y}{\partial z^2}$$

In the conventional approach, this equation is presented to students and immediately followed by common methods of solving for E_y: product solutions, numerical solutions, and so forth. The student who is motivated by surviving a difficult class will, of course, learn these methods; the student also motivated by curiosity, however, would like to know more: What does the equation itself mean? Yes, it describes waves, but can we be more specific?

An instructor familiar with teaching the art of insight will, at that point, introduce an analogy: Instead of E_y, think of a stretched string with a vibrating amplitude y. What does the wave equation mean in that case? The answer is straightforward: The acceleration ($\partial^2 y/\partial t^2$) of any point on the string depends on the phase velocity v_ϕ of the wave and the local curvature ($\partial^2 y/\partial z^2$) of the string at that point. That is, the faster the wave passes, the greater the acceleration; at the same time, an initial condition of amplitude (not wavefront) curvature is required for a wave to propagate.

Further questions from the curious student are then inevitable, such as: What does the phase velocity depend on? Comparing electromagnetic wave propagation in vacuum with that in glass, for example, the frequency ω (and "acceleration" $\partial^2 E_y/\partial t^2 \sim \omega^2$) remain the same; in addition, the wavelength λ in glass is shorter, giving more curvature ($\sim 1/\lambda^2$).

The wave equation then tells us that, if the curvature in glass is higher, the phase velocity must be slower to maintain the same ω—with the refractive index n describing how much slower via $v_\phi = c/n$ for the speed of light in vacuum equal to c. The concept of refractive index is thus essential for understanding not only the static picture of Snell's law of refraction, but also the dynamics of wave propagation.

And like all good analogies, the string-optics analogy can be carried too far. After all, if an elastic medium such as a string is required for mechanical waves, what medium (or "aether") is required for electromagnetic waves? It actually took the optics and physics communities quite a bit of time to answer this question, with the interferometric experiments of Michelson and Morley showing that there is no aether, and that electromagnetic waves self-propagate.

And so, with the inquiry-based motivation of looking for a first-order intuitive understanding of the wave equation, we have discovered a powerful teaching and learning methodology—i.e., the art of insight, or distilling a problem down to its physical essence—that quickly led us in multiple directions, covering a deep conceptual overview of how some optical "things" work.

For more details on this topic and many others, two excellent references are: (1) F. S. Crawford, Jr., *Waves*, McGraw-Hill (1968) and (2) S. Mahajan, *The Art of Insight in Science and Engineering*, MIT Press (2014).