Parametric cost models for space telescopes

H. Philip Stahl, Todd Henrichs, Courtnay Dollinger
PARAMETRIC COST MODELS FOR SPACE TELESCOPES

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I. INTRODUCTION

Multivariable parametric cost models for space telescopes provide several benefits to designers and space system project managers. They identify major architectural cost drivers and allow high-level design trades. They enable cost-benefit analysis for technology development investment. And, they provide a basis for estimating total project cost. A survey of historical models found that there is no definitive space telescope cost model. In fact, published models vary greatly [1]. Thus, there is a need for parametric space telescopes cost models. An effort is underway to develop single variable [2] and multi-variable [3] parametric space telescope cost models based on the latest available data and applying rigorous analytical techniques.

Specific cost estimating relationships (CERs) have been developed which show that aperture diameter is the primary cost driver for large space telescopes; technology development as a function of time reduces cost at the rate of 50% per 17 years; it costs less per square meter of collecting aperture to build a large telescope than a small telescope; and increasing mass reduces cost.

II. MODEL CREATION

To develop a parametric cost models requires data. Cost and engineering data has been collected on 59 different parameters for 23 different UV, optical or infrared space telescopes. (Table 1 and 2)

<p>| Table 1: UV/OIR Cost Model Missions Database |</p>
<table>
<thead>
<tr>
<th>UV/Optical Telescopes</th>
<th>Infrared Telescopes</th>
</tr>
</thead>
<tbody>
<tr>
<td>EUVE</td>
<td>CALIPSO</td>
</tr>
<tr>
<td>FUSE</td>
<td>Herschel</td>
</tr>
<tr>
<td>GALEX</td>
<td>ICESat</td>
</tr>
<tr>
<td>HiRISE</td>
<td>IRAS</td>
</tr>
<tr>
<td>HST</td>
<td>ISO</td>
</tr>
<tr>
<td>HUT</td>
<td>JWST</td>
</tr>
<tr>
<td>IUE</td>
<td>SOFIA</td>
</tr>
<tr>
<td>Kepler</td>
<td>Spitzer (SIRTF)</td>
</tr>
<tr>
<td>Copernicus (OAO-3)</td>
<td>TRACE</td>
</tr>
<tr>
<td>SOHO/EIT</td>
<td>WIRE</td>
</tr>
<tr>
<td>UIT</td>
<td>WISE</td>
</tr>
</tbody>
</table>

Table 2: Cost Model Variables Study and the completeness of data knowledge

<table>
<thead>
<tr>
<th>Parameters</th>
<th>% of Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>OTA Cost</td>
<td>89%</td>
</tr>
<tr>
<td>Total Phase A-D Cost w/o LV</td>
<td>84%</td>
</tr>
<tr>
<td>Aperture Diameter</td>
<td>100%</td>
</tr>
<tr>
<td>Avg. Input Power</td>
<td>95%</td>
</tr>
<tr>
<td>Total Mass</td>
<td>89%</td>
</tr>
<tr>
<td>OTA Mass</td>
<td>89%</td>
</tr>
<tr>
<td>Spectral Range</td>
<td>100%</td>
</tr>
<tr>
<td>Wavelength Diffraction Limit</td>
<td>63%</td>
</tr>
<tr>
<td>Primary Mirror Focal Length</td>
<td>79%</td>
</tr>
<tr>
<td>Design Life</td>
<td>100%</td>
</tr>
<tr>
<td>Data Rate</td>
<td>74%</td>
</tr>
<tr>
<td>Launch Date</td>
<td>100%</td>
</tr>
<tr>
<td>Year of Development</td>
<td>95%</td>
</tr>
<tr>
<td>Technology Readiness Level</td>
<td>47%</td>
</tr>
<tr>
<td>Operating Temperature</td>
<td>95%</td>
</tr>
<tr>
<td>Field of View</td>
<td>79%</td>
</tr>
<tr>
<td>Pointing Accuracy</td>
<td>95%</td>
</tr>
<tr>
<td>Orbit</td>
<td>89%</td>
</tr>
<tr>
<td>Development Period</td>
<td>95%</td>
</tr>
<tr>
<td>Average</td>
<td>88%</td>
</tr>
</tbody>
</table>

Statistical correlations have been evaluated between 19 variables and used to develop single and multi-variable cost estimating relationships (CERs) to model Optical Telescope Assembly (OTA) and Total Mission Cost. CERs are evaluated for their ‘goodness’.

Optical Telescope Assembly (OTA) is defined as the space observatory subsystem which collects electromagnetic radiation and focuses it (focal) or concentrates it (afocal). An OTA consists of the primary mirror, secondary mirror, auxiliary optics and support structure (such as optical bench or truss structure, primary support structure, secondary support structure or spiders, etc.). An OTA does not include science instruments or spacecraft subsystems. Cost is defined as prime contract cost without any NASA labor or overhead. Total mission cost is defined as Phase A-D cost, excluding: launch cost; costs associated with NASA labor (civil servant or support contractors) for program management, technical insight/oversight; or any NASA provided ground support equipment, e.g. test facilities. Accounting for NASA overheads would increase the cost by at least 10% and maybe as much as 33%.

Goodness of a Fit or a Correlation is tested via a range of statistical measures, including Pearson’s $$r^2$$ coefficient, Student T-Test p-value and standard percent error (SPE). Pearson’s $$r^2$$ (typically denoted as just $$r^2$$) describes the
percentage of agreement between the model and the actual cost. For multi-variable models, we use Adjusted Pearson's $r^2$ (or $r^2_{adj}$) which accounts for the number of data points and the number of variables. In general, the closer $r^2$ (or $r^2_{adj}$) is to 1.0 or 100%, the better the model. SPE is a normalized standard deviation of the fit residual (difference between data and fit) to the fit. The closer SPE is to 0, the better the fit. Please note that since SPE is normalized, a small variation divided by a very small fit value can yield a very large SPE. The p-value is the probability that a fit or correlation would occur if the variables are independent of each other. The closer the p-value is to 0, the more significant the fit or correlation. The closer it is to 1, the less significant. If the p-value for a given variable is small, then removing it from the model would cause a large change to the model. If it is large, then removing the variable will have a negligible effect. Also, it is important to consider how many data points are included in a given correlation or fit.

Table 3 summarizes the cross-correlation between specific key parameters and Total Mission Cost, OTA Cost and OTA Areal Cost (where areal cost is defined as OTA Cost divided by OTA collecting area). For each parameter, Table 3 reports its correlation to cost, the correlation's p-value and the number of data points in the correlation. Diameter appears to be the most significant cost driver. So, in addition to Total Cost and OTA Cost we have examined OTA Areal Cost, i.e. OTA Cost per unit Area of Primary Mirror collecting aperture. Diameter is correlated with all three with a significance of greater than 99%. Primary Mirror Focal Length is also a significant correlation, but it is multi-collinear with Diameter. The assumed explanation is that all space telescopes tend to have the same basic PM F/#. Pointing Accuracy has reasonable correlation with cost. And, as expected from engineering judgment, it has significant correlation (99% confidence level) with diameter and OTA mass. Interestingly, pointing is not multi-collinear with either. As expected, Total Mass correlates most significantly with Total Cost while OTA Mass correlates most significantly with OTA Cost. Unexpectedly, Minimum Spectral Range Value and Operating Temperature do not have a significant correlation with any Cost. However, Spectral Minimum does have a role in multi-variable cost models. As expected Electrical Power, Design Life and Development Period have significant correlations (99% confidence) with Total Cost. Also unexpected is that TRL and Launch Year do not have significant correlations. But, they both have roles in multi-variable cost models. One problem with TRL is that there are only 8 data points. Also, it is a qualitative and not a quantitative parameter.

### III. COST MODELS

Four single variable cost estimating relationships (CERs) have been developed for OTA Cost and Total Mission Cost as a function of OTA diameter, OTA mass and total mission mass [2]. These models were developed with and without JWST. The benefit of including JWST is that it is the most current mission. The disadvantage is that its cost is not yet final. For the purpose of this paper, we will include the 2009 JWST C/D final cost estimate. In general, including JWST does affect the model $r^2_{adj}$ but does not increase the noisiness of the fit as represented by the SPE. Additionally, these models are developed only for free-flying missions. Of the 23 missions in the data base, there are 19 free flying telescopes (17 for which we have OTA cost data) and 4 that are attached (3 to the Space Shuttle Orbiter and SOFIA to a Boeing 747 airplane). As will be discussed below with regard to mass models, attached missions have a significantly different cost dependency than free-flying missions. Therefore, we excluded attached missions from the models.

Engineering judgment says that OTA Cost is most closely related to OTA engineering parameters. But, managers and mission planners are more interested in Total Phase A-D Cost. Analysis of the 14 free-flying missions for which we have both OTA cost data and Phase A-D Total Mission cost data indicates (Fig 1) that OTA cost is ~20% of total mission cost ($R^2 = 96\%$) with a model residual standard deviation of approximately $\$3000$. It is interesting to note that there is significant variation in this percentage for small missions but not for large. Additionally, we created a
common Work Breakdown Structure (WBS) and mapped onto it the individual WBSs of 7 missions (including HST and JWST) for which we had detailed cost data. This analysis indicates that OTA cost is 30% of Total (Fig 2).

Fig 1: Total Mission Cost vs Percentage that OTA Cost is of Total Cost.

Fig 2: Average WBS cost allocation for 7 free flying UV/OIR systems.

Fig 3 plots OTA Cost for free-flying space telescopes as a function of Primary Mirror Diameter. The regression fit for this data is:

$$\text{OTA Cost} \sim \text{Aperture Diameter}^{1.2} \quad (N = 17; r^2 = 75\%; \text{SPE} = 79\%) \text{ with 2009 JWST}$$

Note that the Chandra data point is for reference only. It is not included in the regression. And, it is plotted based upon the equivalent normal incidence mirror diameter it would have if all of its x-ray mirrors were unrolled.

Given that the OTA Cost might be dominated by the large apertures for HST and JWST, a model was also created for normalized Areal OTA Cost (Fig 4):

$$\text{OTA Areal Cost} \sim \text{Aperture Diameter}^{-0.74} \quad (N = 17; r^2 = 55\%; \text{SPE} = 78\%) \text{ with JWST}$$

A key finding of this analysis is that Areal Cost decreases with aperture size. It is less expensive per photon to build a large aperture telescope than a small aperture telescopes. Large aperture telescopes provide a better ROI.

Fig 3: OTA Cost vs Aperture Diameter scaling law for 17 free flying UV/OIR systems (including 2009 JWST). Plot includes 90% confidence and prediction intervals, and data points. Chandra data is not in the regression.

Fig 4: OTA Areal Cost vs Aperture Diameter scaling law for 17 free flying UV/OIR systems (including 2009 JWST). Plot includes 90% confidence and prediction intervals, and data points. Chandra is not in the regression.

From an engineering and a scientific perspective, aperture is the best parameter to build a space telescope cost model. Aperture defines the observatory’s science performance and determines the payload’s size and mass. And, while the results are consistent with some historical cost models, our results invalidate long held ‘intuitions’ which are often purported to be ‘common knowledge’. Space telescope costs vary almost linearly with diameter and not to a power of 1.6X or 2.0X or even 2.8X. But, a model based on diameter alone has only a ~75% agreement with the OTA cost data and ~55% agreement with the OTA areal data. Therefore, a multi-variable step wise regression is required to look for other factors which influence cost. First, one performs a two variable regression of Diameter
plus each of the other parameters and evaluates the statistical ‘goodness’ of each regression (Fig 5). Once a good two variable model is selected, the process can be repeated to add a third variable.

![Fig 5: Two variable regression for OTA Cost vs Aperture Diameter and a 2nd Variable](image)

Regarding potential two variable OTA Cost models, three parameters have significance greater than 98%: TRL, Year of Development (YoD) and Launch Year (LYr). The Diameter + TRL model has a slightly higher $r^2_{adj}$ than the other models, but it also has a high SPE. This may be because of the relatively few TRL data points in our data base. Or, it may be because TRL value is subjective and thus has a natural ‘fuzziness’ to its data values. Based on coefficient significance, other parameters of potential interest are Field of View (82%), OTA Mass (74%), OTA Areal Density (74%), Power (77%) and Data Rate (72%). But all, except Data Rate, do not simultaneously increase $r^2_{adj}$ and decrease SPE. And, some, such as FOV, are particularly poor. It should also be noted that OTA Mass is multicollinear with Aperture Diameter – which only makes sense, i.e. the larger the telescope, the more mass it should have. Therefore, mass is not a good second variable candidate.

Both YoD and LYr have similarly high $r^2_{adj}$ values and significantly lower SPE values. And, if you round significant digits, each model is virtually identical:

$$\text{OTA Cost} \sim D^{1.34} e^{-0.04(\text{ LYr}-1960)}$$

$$\text{(N = 17, } r^2_{adj} = 93\%; \text{SPE = 39%)}$$

$$\text{OTA Cost} \sim D^{1.27} e^{-0.04(\text{ YoD}-1960)}$$

$$\text{(N = 16, } r^2_{adj} = 95\%; \text{SPE = 39%)}$$

Launch Year has the advantage of being a definite date, but it has the disadvantage that a launch can be delayed. However, while a launch delay tends to increase the Total Mission Cost, it may not increase OTA Cost. Year of Development yields a slightly better regression, but its exact date is subject to definition. Does it start with Phase A or Phase C? Regardless of which parameter is used, the message is clear: technology improvements reduce OTA Cost as a function of time by approximately 50% every 17 years. For completeness, a two variable OTA Areal Cost regression yielded the same basic results.

The next step is to try adding a third parameter. For our data base of free-flying missions, two different regressions were preformed for OTA Cost versus Diameter, a ‘year’ parameter and each of the other variables as the third parameter. Neither regression yielded a satisfactory model. Next, we decided to add some wavelength diversity by including missions with shorter and longer wavelengths. Specifically, we added WMAP, TDRS-1, TDRS-7, EUVE, Chandra and Einstein. With the extra missions, two satisfactory three variable models were achieved:

$$\text{OTA Cost} \sim D^{1.5} \lambda^{-0.17} e^{-0.03(\text{YoD}-1960)}$$

$$\text{(N = 20, } r^2_{adj} = 92\%; \text{SPE = 76%)}$$

$$\text{OTA Cost} \sim D^{1.05} \lambda^{-0.13} e^{-0.03(\text{LYr}-1960)}$$

$$\text{(N = 23, } r^2_{adj} = 63\%; \text{SPE = 69%)}$$

Finally, while aperture is the single most important parameter driving science performance, system mass determines what vehicle can be used to launch it. Also, significant engineering costs are expended to keep a given payload inside of its allocated mass budget, including light-weighting mirrors and structure. Therefore, mass is a potential important CER.

Fig 6 plots Total Cost vs Total Mission Mass for 15 free-flying missions. The regression of this data is:
**Total Cost ~ Total Mass**\(^{1.12}\) \((N = 15; r^2 = 86\%; \text{SPE} = 71\%)\) with JWST

Fig 7 plots OTA Cost vs OTA Mass for both free-flying and attached missions. The regression for only the free-flying missions is:

**OTA Cost ~ OTA Mass**\(^{0.72}\) \((N = 15; r^2 = 92\%; \text{SPE} = 93\%)\) with JWST

While OTA Mass may appear to be a good indicator of OTA Cost because it has the highest Pearson's \(r^2\), it also has the highest SPE. And, please note that just because we have created a mass CER, we do not recommend using it. In general mass should be avoided as a CER because it is a secondary indicator. Mass depends upon the size of the telescope. Bigger telescopes have more mass. And, bigger telescopes require bigger spacecraft and bigger science instruments which require more power — all which adds mass. And, because many missions are designed to a mass-budget defined by launch vehicle constraints, the result can be a very complex, risky, and expensive mission architecture when trying to extend the state-of-the-art in either wavelength or aperture. This effect can be seen in Fig 6 where JWST has nearly half the total mass of HST but still has a higher total mission cost — because JWST is bigger and more complex than HST. But, this does not have to be the case.

As indicated in Fig 7 and Fig 8, it is possible to reduce cost by building space telescopes with different design rules. Fig 7 shows that Attached OTAs have a different cost versus mass relationship than free-flying OTAs. The reason is that 'attached' OTAs have a much more relaxed mass budget constraint than 'free-flying' OTAs. Fig 8 shows two key findings. First, the OTA cost per kilogram is entirely different for free-flying versus attached missions. Attached OTAs are approximately 5.5X less expensive per kg than free-flying OTAs. Second, the cost per kg for these classes of missions is independent of aperture size. Other analysis shows that for a given aperture size, attached OTAs are on average ~2X more massive and ~2.5X less expensive than free-flying OTAs. Finally, there may be a third cost class – Planetary – but we are not certain because HiRISE is our only planetary OTA data point.

The importance of these findings is that they invalidate the ‘common assumption’ that the more massive the mission, the more expensive the mission. The only reason that more massive missions are more expensive is because they have more ‘stuff’. When one compares missions with similar performance properties, it is less expensive to design, build and fly a simple mission with more mass than a lightweight complex mission. Therefore, maybe the best way to reduce the cost of future large aperture space telescopes is to develop cost effective heavy lift launch vehicles which will enable mission planners to trade complexity for mass.

**IV. CONCLUSIONS**

Cost models are invaluable for system designers. They identify major architectural cost drivers and allow high-level design trades. They enable cost-benefit analysis for technology development investment. And, they provide a basis for estimating total project cost. A study is in-process to develop single and multivariable parametric cost model for space telescopes. Cost and engineering parametric data has been collected on 30 different missions and extensively analyzed for 23 normal incidence UV/OIR space telescopes. Statistical correlations have been developed for 19 of the 59 variables sampled.
From an engineering & science perspective, Aperture Diameter is the best parameter for a space telescope cost model. But, the single variable model only predicts 75% of OTA Cost:

\[
\text{OTA Cost} \sim D^{1.2} \quad (N = 17; \ r^2_{\text{adj}} = 75\%; \ \text{SPE}=79\%) \text{ with 2009 JWST}
\]

Two and three variable models provide better estimates:

\[
\begin{align*}
\text{OTA Cost} & \sim D^{1.3} e^{-0.04(LYr-1960))} \\
& (N = 17, \ r^2_{\text{adj}} = 93\%; \ \text{SPE}=39\%)
\end{align*}
\]

\[
\begin{align*}
\text{OTA Cost} & \sim D^{1.3} e^{-0.04(YoD-1960))} \\
& (N = 16, \ r^2_{\text{adj}} = 95\%; \ \text{SPE}=39\%)
\end{align*}
\]

\[
\begin{align*}
\text{OTA Cost} & \sim D^{1.15} \lambda^{-0.17} e^{-0.03(YoD-1960))} \\
& (N = 20, \ r^2_{\text{adj}} = 92\%; \ \text{SPE} = 76\%)
\end{align*}
\]

where: \( D = \) Aperture Dia, \( LYr = \) Launch Yr, \( YoD = \) Yr of Development, and \( \lambda = \) Spectral Min Wavelength.

At present the study has not yet produced a satisfactory model for Total Mission Cost.

While mass does yield a statistically significant regression which implies that more massive telescopes cost more, this finding is artificial, misleading, could easily lead one to make inappropriate programmatic decisions, and it contradicts the fact that JWST costs more than HST but has half the mass. A careful study of the data actually indicates that for any given aperture diameter, attached OTAs are on average 2X more mass and 2.5X less expensive than free-flying OTAs; the cost per kilogram of attached OTAs is \(~5.5X\) lower than for free-flying OTAs; and that the cost per kg of these two ‘design rule’ classes is independent of aperture. Finally, there may be a third even more expensive ‘design rule’ class – Planetary OTAs – but we only have one data point currently in the data base.

The primary conclusions of the cost modeling study to date are:

- The primary cost driver for Space Telescope Assemblies is Aperture Diameter.
- It costs less per collecting area to build a large aperture telescope than a small aperture telescope.
- Technology development as a function of time reduces cost at the rate of 50% per 17 years.
- If all other parameters are held constant, adding mass reduces cost and reducing mass increases cost.

REFERENCES

