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HYBRID ARQ COMMUNICATIONS FOR SEVERELY DEGRADED HERMITE-GAUSSIAN FSO LINK

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I. ABSTRACT

In this paper, the potential of hybrid automatic-repeat request (HARQ) algorithm in enhancing the performance of a free-space optical (FSO) communication system, which is impaired by weak turbulence and non-negligible residual pointing error, is explored. In particular, the FSO system is assumed to use orthogonal Hermite-Gaussian (HG) beam patterns to exploit spatial multiplexing of optical radiation. Furthermore, it is assumed that a direct-detection mechanism is utilized at the receiver and that the optical radiation is modulated using pulse-position modulation (PPM). Via analytical results and numerical analysis, it is demonstrated that HARQ not only overcomes the impact of spatial error, but also offers a means of achieving near error free communications provided that the delay associate with HARQ can be tolerated. In general, HARQ offers several orders of magnitude improvement in performance.

II. INTRODUCTION

Free-space optical communication systems have been subject of numerous studies for applications ranging from imaging to high-speed communications. FSO systems are required to achieve high data rates in the presence of atmospheric effects and other impairments unique to optical systems. Once major hurdle that one has to overcome is the requirement to establish precise pointing before a communications link can be established. The other challenge is to explore techniques to achieve high capacity communications. To address the latter problem, in recent years, researchers have considered spatial multiplexing using Hermite-Gaussian modes, for instance, see [1] and pp. 238 in [2]. Spatial pointing error, largely due to platform vibrations and atmospheric induced beam wander [3], pp. 146, is yet another challenge for FSO communications. As noted above, this problem must be addressed prior to establishing a communications link. The direct impact of pointing error is the degradation of the received signal level, which in turn reduces the ability of the link to establish high data rates, for example, see [4]–[12]. In recent years, the concept of HARQ for FSO has gained some attention. This concept was first introduced for FSO applications in [13]. In particular, the use of chase-combining (CC) in HARQ was shown to result in several orders of magnitude improvement in performance when compared with the standard FSO systems.

This paper addresses the problem of HARQ in HG FSO systems subject to spatial error. The impact of spatial error is of particular importance to the HG systems which rely on almost perfect spatial acquisition in order to maintain orthogonality among the HG modes, thereby increasing capacity. In a recent study, the impact of residual pointing error on the behavior of HG optical beam was studied [14]. In particular, it was shown that the presence of spatial error significantly distorts the HG modes, resulting in the cross-talk among the orthogonal HG modes and the loss of useful energy for a given HG channel. Such losses will ultimately limit the potential capacity of HG-FSO systems. In the present study, we focus on providing the performance improvement one could expect in HG FSO systems subject to pointing error when HARQ techniques is brought to bear. Finally, it is important to note that the spatial error assumes a Gaussian statistics. It is noteworthy that for efficient estimators, such as maximum-likelihood (ML) estimators, the estimates of the spatial error are either efficient or asymptotically-efficient [15]. Hence, the assumption of a spatial error with Gaussian statistics is a reasonable assumption.

III. HG BEAM PROFILE

We assume a Hermite-Gaussian beam profile, which is a generalized eigenfunction of the optical field equation. Specifically, the optical field intensity[1] can be expressed as [2], pp. 238,

$$I_{l_x l_y}(x,y) = I_0 \exp\left(-\frac{2x^2}{W^2} - \frac{2y^2}{W^2}\right) H_{l_x}^2\left(\frac{\sqrt{2}x}{W}\right) H_{l_y}^2\left(\frac{\sqrt{2}y}{W}\right) \quad (1)$$

where (x, y) denotes the Cartesian coordinates of a point in the transversal receiver plane, $I_{l_x l_y}(x,y)$ is the intensity at (x, y), I_0 is the field intensity at $x = y = 0$, and $H_{l_x}(x)$ is the Hermite polynomial of order l_x. Furthermore, W denotes the beam waist at the receiver plane[2]. Note that, for $l_x = l_y = 0$, the above reduces to the standard Gaussian beam profile

[1]Note that we are interested in the field intensity and not the complex optical field.

[2]Although W may not be the beam waist of a Generalized HG profile, where the intensity must drop to e^{-2} of its peak value at the origin, the term ”waist” is used here in order to be consistent with the literature regarding Gaussian profile.

often used in FSO systems. The above formulation allows for the separation of x and y coordinate components. That is, $I_{l_x l_y}(x,y) = I_{l_x}(x) I_{l_y}(y)$.

In this paper, it is assumed that $l_x = l_y = N$ where N is the number of modes in HG system. This implies that N^2 simultaneous spatial channels can be accommodated in this formulation.

IV. HG-HARQ

In [13], the performance of a HARQ PPM system was studied. For the sake of completeness, excerpts of the previous result is outlined below. If $P_E^{CH}(m|\eta_m)$ denotes the conditional (conditioned on channel states) probability of bit error after m retransmissions (with a maximum of M retransmissions) with equally-likely messages, we then have

$$P_b^{CH}(m|\eta_m) = Q\left(\frac{\beta\eta_m}{\sqrt{\alpha(\eta_m + 2mp_{ba}) + 2m\sigma_n^2}}\right) \tag{2}$$

where $Q(x) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \exp\left(-\frac{x^2}{2\sin^2(\theta)}\right) d\theta$, $\eta_m = \sum_{j=1}^{m} p_{s_j}$, $\beta = \frac{RGT_b}{2}$ with R, G, and $T_b/2$ denoting the responsivity of the detector in A/W, the gain of the detector/filtering stage, and the slot duration (half the bit duration) in sec, respectively. Furthermore, $\sigma_n^2 = \frac{KT^o T_b}{R_L}$ denotes the variance of the thermal noise with K, T^o, and R_L denoting the Boltzmann's constant, the receiver temperature in Kelvin, and photo-detector load resistance in Ω, respectively. p_{s_j} is the received optical power for the jth retransmission while p_{ba} denotes the received optical power due to background radiation. Also, $\alpha = \frac{eRG^2T_b}{2}$ with e denoting the charge of an electron. As noted in [13], the pdf of η_m, which is the sum of lognormal random variables (clear-air turbulence), is needed. In a recent study [16], an accurate approximation for the pdf of η_m was suggested. In particular, for small m (as is the case here), the result in [16] has been shown to be fairly accurate [17]. Hence, $P_b^{CH}(m)$, the bit error rate for m retransmissions, is given by

$$P_b^{CH}(m) = \int_0^{\infty} P_b^{CH}(m|y) f_{\eta_m}(y) dy \tag{3}$$

where $f_{\eta_m}(y)$ denotes the pdf of η_m. Since it is assumed that each packet is utilizing a forward-error-correction (FEC) algorithm, which can correct up to t errors, and that HARQ is applied in each step of the M retransmissions (i.e., the metric is recomputed in each retransmission), then the probability of incorrectly recovering a packet after M retransmissions using HARQ is

$$P_{pac}^{CH}(M) = \prod_{m=1}^{M} \left(1 - \sum_{q=0}^{t} \binom{L}{q} P_b^{CH}(m)^q \left[1 - P_b^{CH}(m)\right]^{L-q}\right) \tag{4}$$

In the event that HARQ is not performed (namely, ARQ is employed to determine if a packet has been received correctly), the probability that a packet is incorrectly recovered after M retransmissions is

$$P_{pac}(M) = \left(1 - \sum_{q=0}^{t} \binom{L}{q} P_b^q \left[1 - P_b\right]^{L-q}\right)^M \tag{5}$$

where $P_b = P_b^{CH}(1)$ is the special case of Chase combining with one transmission ($m = 1$).

V. IMPACT OF SPATIAL ERROR

The analytical results presented above can be applied to each spatial channel of an HG system in the absence of spatial tracking error. Since HG spatial modes are orthogonal, the above result can be directly extended to multiple spatial channel type system. However, if one encounters non-negligible spatial tracking error, there will be cross-talk between the spatial channels [14].

The cross-talk appears as a loss in the useful signal power while there will be an interference component to the noise power. We consider $N = 3$ and, without the loss of generality, assess the performance of HG channel $l_x = l_y = 1$.

Let $x_{e,nz}$ and $y_{e,nz}$ denote the normalized (normalized by W) residual spatial errors in the x and y directions, respectively, in the detector plane. The normalized cross-talk per dimension can be evaluated using

$$\Gamma_{cs,l_p,l_q}\left(x_{e,nz}\right)= \left\{\int_{-\infty}^{\infty} H_{l_p}\left(\sqrt{2}x_{nz}\right) H_{l_q}\left(\sqrt{2}\left(x_{nz}-x_{e,nz}\right)\right) \times e^{-\left(x_{nz}-x_{e,nz}\right)^2-x_{nz}^2} dx_{nz}\right\} / \left\{\left(\int_{-\infty}^{\infty} H_{l_p}^2\left(\sqrt{2}x_{nz}\right) \times e^{-2x_{nz}^2} dx_{nz}\right)^{0.5} \int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} H_{l_q}^2\left(\sqrt{2}x_{nz}\right) \times e^{-2x_{nz}^2} dx_{nz}\right)^{0.5}\right\};\ l_p \neq l_q. \tag{6}$$

Furthermore, the signal level degradation per dimension is given by

$$\Gamma_{l_p,s}\left(x_{e,nz}\right)= \left\{\int_{-\infty}^{\infty} H_{l_p}\left(\sqrt{2}x_{nz}\right) H_{l_p}\left(\sqrt{2}\left(x_{nz}-x_{e,nz}\right)\right) \times e^{-\left(x_{nz}-x_{e,nz}\right)^2-x_{nz}^2} dx_{nz}\right\} / \int_{-\infty}^{\infty} H_{l_p}^2\left(\sqrt{2}x_{nz}\right) \times e^{-2x_{nz}^2} dx_{nz}. \tag{7}$$

Hence, we need to replace p_{s_j} with $p_{s_j}(x_{e,nz}, y_{e,nz})$, which is now the actual signal power that is captured by the receiver in the presence of spatial error for the jth retransmission. In [14], the impact of spatial error for each dimension is shown to be

$$\Gamma_{l_p,s}\left(x\right)= \begin{cases} -e^{-\frac{x^2}{2}}\left(-1+x^2\right) & l_p=1 \\ \frac{1}{2}e^{-\frac{x^2}{2}}\left(2-4x^2+x^4\right) & l_p=2 \\ -\frac{1}{6}e^{-\frac{x^2}{2}}\left(-6+18x^2-9x^4+x^6\right) & l_p=3 \end{cases} \tag{8}$$

Furthermore, the impact of cross-talk (we assume only 3 HG layers for this analysis) can be assessed for each dimension using (see [14])

$$\Gamma_{cs,l_p,l_q}\left(x\right)= \begin{cases} -\frac{\sqrt{2}}{2}e^{-\frac{x^2}{2}}x\left(-2+x^2\right) & \begin{matrix} l_q=1 \\ l_p=2, \end{matrix} \\ -0.408e^{-\frac{x^2}{2}}x^2\left(-3+x^2\right) & \begin{matrix} l_q=1 \\ l_p=3, \end{matrix} \\ 0.288e^{-\frac{x^2}{2}}x\left(6-6x^2+x^4\right) & \begin{matrix} l_q=2 \\ l_p=3 \end{matrix} \end{cases}. \tag{9}$$

The above (see [14]) implies that $p_{s_j}(x_{e,nz}, y_{e,nz})$ for the case of $l_x = l_y = 1$ is given by

$$p_{s_j}(x_{e,nz}, y_{e,nz}) = p_{s_j}\Gamma_{1,s}\left(x_{e,nz}\right)\Gamma_{1,s}\left(y_{e,nz}\right) \tag{10}$$

Further, mp_{ba} for layer $l_x = l_y = 1$ in eq. (2) is replace as follows:

$$2mp_{ba} \rightarrow 2mp_{ba} + I_1, \tag{11}$$

where

$$\begin{aligned} I_1 = \textstyle\sum_{j=1}^{m} p_{s_j}\, \{&\Gamma_{cs,3,1}\left(x_{e,nz}\right)\Gamma_{cs,3,1}\left(y_{e,nz}\right)+ \\ &\Gamma_{cs,3,1}\left(x_{e,nz}\right)\Gamma_{cs,2,1}\left(y_{e,nz}\right)+ \\ &\Gamma_{cs,3,1}\left(y_{e,nz}\right)\Gamma_{cs,2,1}\left(x_{e,nz}\right)+ \\ &\Gamma_{cs,2,1}\left(y_{e,nz}\right)\Gamma_{cs,2,1}\left(x_{e,nz}\right)+ \\ &\Gamma_{cs,2,1}\left(y_{e,nz}\right)\Gamma_{1,s}\left(x_{e,nz}\right)+ \\ &\Gamma_{cs,2,1}\left(x_{e,nz}\right)\Gamma_{1,s}\left(y_{e,nz}\right)+ \\ &\Gamma_{cs,3,1}\left(x_{e,nz}\right)\Gamma_{1,s}\left(y_{e,nz}\right)+ \\ &\Gamma_{cs,3,1}\left(y_{e,nz}\right)\Gamma_{1,s}\left(x_{e,nz}\right)\ \} \end{aligned} \tag{12}$$

In this formulation, it is assumed that the pointing error remains constant for m consecutive retransmissions. Since we are considering Gbps data rates, and the pointing error has a correlation time of tens of msec, this assumption is quite accurate. This results point to the direct impact of cross-talk on the performance of an HG system. In arriving at this results, it is assumed that the interfering signal is a PPM signal. In this formulation, it is assumed that the power in the jth retransmission is the same across all layers. The above result is due to the fact that the presence of modulated cross-talk from multiple layers may be modeled (this approximation becomes less valid when there are small number of channels) as a Gaussian random variable with zero-mean and a variance of I_n. Hence, the contribution of cross-layer interference is an increase in the Gaussian background noise power by I_n. The worst case scenario is when spatial errors in each dimension reaches its maximum allowable range. We consider several scenarios here. Since the approximations states to arrive at the results above must be taken into account, we consider $0.05 \leq x_{e,nz}=y_{e,nz} \leq 0.35$ as the range of the spatial error [14].

VI. Numerical Results

Similar to the study in [13], we assume M=2 and 3. Further, we assume that $t = 1$ (single error correcting FEC), $L = 128$, bit rate of 1 Gbps, $R_L = 50\ \Omega$, $G = 200$, $R = 0.62$, p_{ba}=-70 dBm, $\sigma^2_{sc} = 0.75$ (weak turbulence), and $T_0 = 300$ K (day-time operation). This allows one to compute the required transmitter power using only geometric and system losses. Also, the x-axis reflects p_s. Without the loss of generality, we consider 3 layers of HG. That is, $N = 3$. For the benefit of providing a fair comparison (see [13]), the x-axes of the plots are scaled so that they depict the total power required in M transmissions. The motivation here is that via retransmission we achieve superior performance when the total transmitted power is the same for HARQ and standard (no-HARQ) systems. In Figs. 1-4, the performances of HARQ and standard systems are compared for normalized spatial errors in the x and y directions of $0.05 - 0.35$. First, we noticed significant degradation in performance in the presence of spatial error. Furthermore, and perhaps more significantly, the use of HARQ allows one to achieve almost error free communication in the presence of spatial error. This is a significant result in that the FSO systems which are exposed to substantial platform vibrations and atmospheric wander and/or tilt can still be operated at a reasonable power level without overburdening the pointing and tracking mechanism. The key conclusion of these numerical results are that the spatial error has a significant impact on the performance of FSO systems and that, with the aid of HARQ, one can still achieve almost error free transmission in the presence of spatial error.

VII. Conclusions

In this paper, the performance of HG FSO systems subject to severe pointing error was studied. It was shown that the use of HARQ can enhance the performance of HG FSO systems by several orders of magnitude. More importantly, in the presence of substantial pointing error, HARQ was shown to yield significant improvement in performance using as little as 3 retransmissions. In fact, one is able to achieve near error free communications if the delay associated with the small number of retransmissions can be tolerated. For near-earth or deep-space applications where performance is of a greater concern than the data rate, this approach offers a viable solution to FSO communications in venues which are hostile to achieving a high degree of pointing accuracy.

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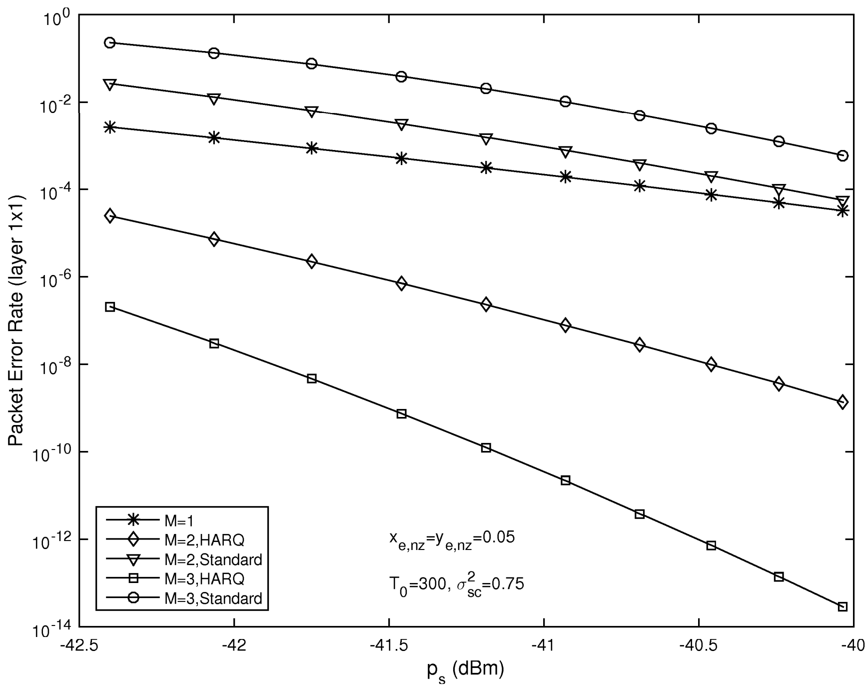

Fig. 1. The performances of HARQ and standard receivers for a 9 layer system. Performance is depicted for layer 1x1. Spatial error is assumed to be $x_{e,nz} = y_{e,nz} = 0.05$.

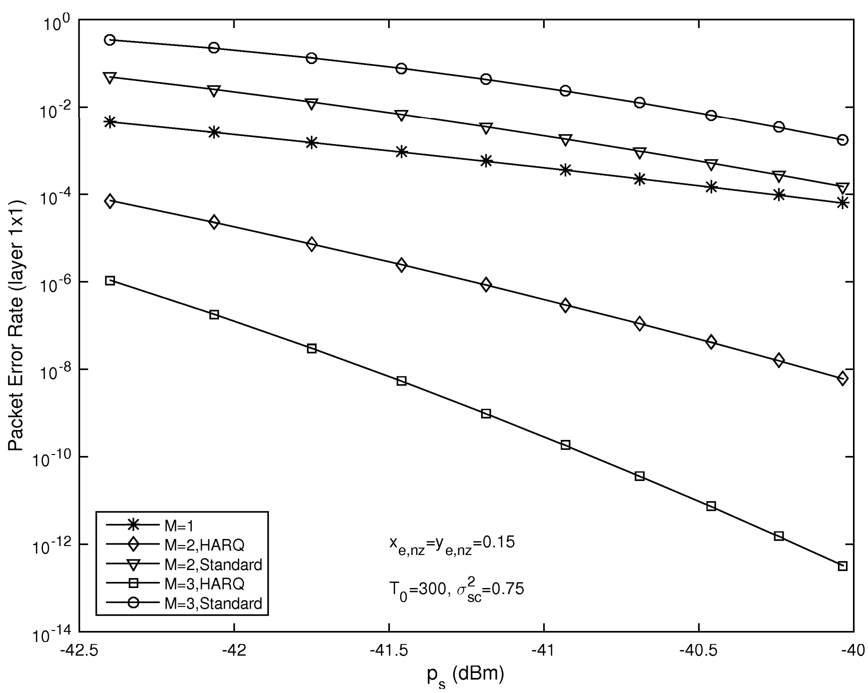

Fig. 2. The performances of HARQ and standard receivers for a 9 layer system. Performance is depicted for layer 1x1. Spatial error is assumed to be $x_{e,nz} = y_{e,nz} = 0.15$.

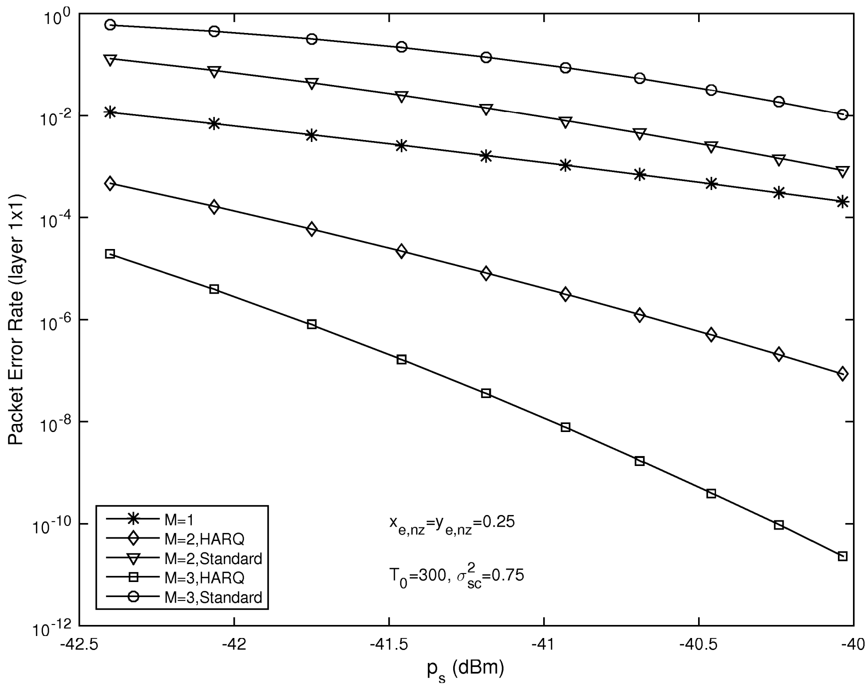

Fig. 3. The performances of HARQ and standard receivers for a 9 layer system. Performance is depicted for layer 1x1. Spatial error is assumed to be $x_{e,nz} = y_{e,nz} = 0.25$.

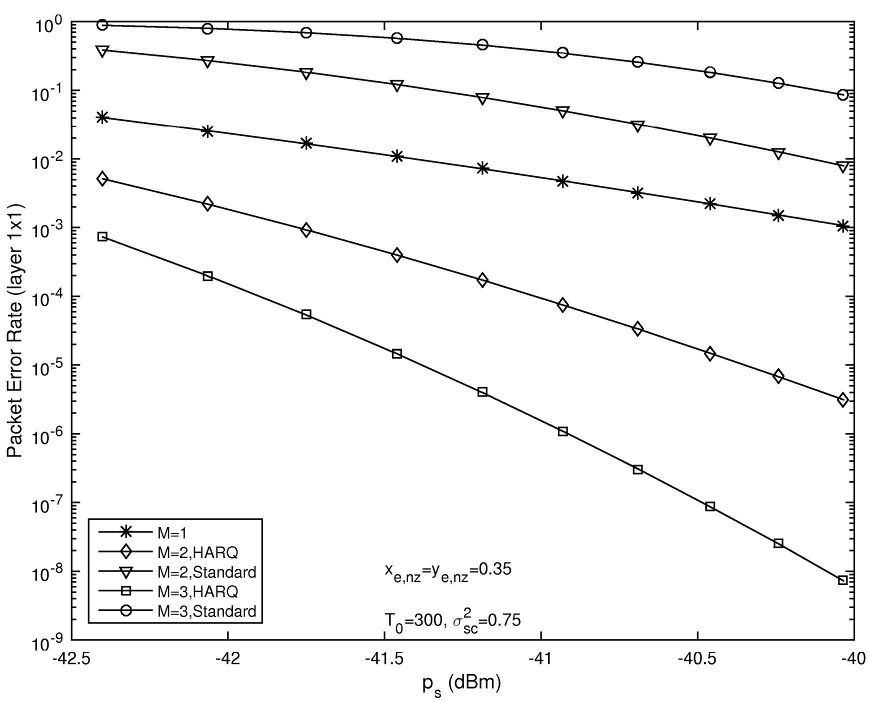

Fig. 4. The performances of HARQ and standard receivers for a 9 layer system. Performance is depicted for layer 1x1. Spatial error is assumed to be $x_{e,nz} = y_{e,nz} = 0.35$.

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