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Ch. Latry

J. M. Delvit

C. Thiebaut



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SENSITIVITY ANALYSIS OF PHASE DIVERSITY TECHNIQUE FOR HIGH RESOLUTION EARTH OBSERVING TELESCOPES

Ch Latry¹, J. M. Delvit¹, C. Thiebaut¹
¹Centre National d'Etudes Spatiales, Toulouse, France
E-mail : christophe.latry@cnes.fr

I. INTRODUCTION

Earth observing systems resolution is directly linked to the telescope diameter through the diffraction. Getting close to a diffraction limited telescope means low level aberrations, with a typical threshold value of $\lambda/30$ for the Wave Front Error (WFE) value. Such a target becomes more and more difficult to achieve for large diameter values. A way of relaxing the realization constraints could be the introduction of some kind of active optics, able to compensate not only for defocus which is already the case for most observation satellites, but also higher order aberrations. The main change would mainly be the possibility of an on board closed loop for WFE assessment and aberration compensation. In order to prepare the next generation of earth observation satellites, CNES has developed the OTOS framework that includes technological developments and optical breadboards, with a significant activity devoted to these active optics principles, and more specifically the WFE assessment technique.

Within the OTOS framework, two well known WFE retrieval techniques have been studied, namely Shack-Hartmann and Phase diversity techniques.

This paper focuses on the phase diversity technique.

CNES has realized a full scale performance analysis with an internally developed algorithm, in order to assess the sensitivity to key parameters.

The aim of this paper is to present a synthesis of this benchmark and will address the following points :

- Introduction of the context of active optics for high resolution earth remote sensing systems
- Principles of phase Diversity technique
- Brief presentation of CNES solving algorithm
- Benchmark presentation
- Synthesis of major results

II. ACTIVE OPTICS FOR EARTH OBSERVING SYSTEMS

A. Optics and image resolution

Looking to the past 30 years of earth remote sensing history reveals a clear trend to spatial resolution improvement. Satellites that have been developed by CNES, the french spatial agency, have not departed from this general tendency, starting from 10m with SPOT1 (1986), improving up to 2.5m with SPOT5 supermode (2002) and making another significant breakthrough with Pléiades-HR satellites and its panchromatic 70cm images, launched in 2011 and 2012.

Next generation should target the 20cm-30cm range.

From an image quality point of view, a remote sensing system is a low pass filter that only retrieves a low spatial frequency band, with a cut-off frequency f_c defined by $f_c = \frac{D}{\lambda}$ in rad^{-1} unit, where D stands for the telescope diameter and λ the wavelength.

Ultimate resolution is thus proportional to D for a given λ . However, effective resolution is lower than this limit because frequency component weakened by the Modulation Transfer Function (MTF), has to be greater than noise level in order to be retrieved. The ratio of Nyquist frequency $\frac{f_e}{2}$ to cut-off frequency f_c , $R = \frac{f_e}{2f_c}$, is thus usually significantly lower than unity and the maximum value depends on Signal to noise ratio.

As pointed out by Fig. 1, the historical evolution of this ratio is to get closer to one, which corresponds to a better use of telescope potential: CNES next generation satellite could have a 70% ratio value.

Since $f_e = \frac{H}{p_e}$ with p_e =ground sampling interval, we may write : $R = \frac{H}{p_e} \times \frac{\lambda}{2D}$ or

$$D = \frac{H}{p_e} \times \frac{\lambda}{2R}$$

With $\lambda=0.65\mu\text{m}$, $H=700\text{km}$, $R=0.7$, the previous formula yields a 1.3m telescope diameter in order to reach resolution $p_e=25\text{cm}$ resolution. It would have been 1.82m using the Pléiades R value (0.5).

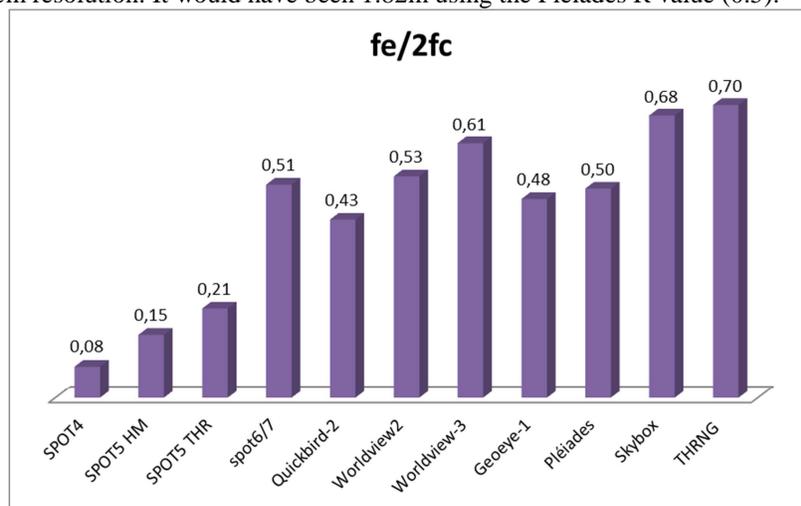


Fig. 1 Historical evolution of Nyquist to cutoff frequency ratio

In order to minimize diameter for a target resolution, MTF has to be maximized in order to use larger R values. Optics have to be as closed as possible to the ideal hypothesis: a diffraction limited telescope. In other words, optics aberrations have to be kept under a very low threshold.

B. Optical aberrations

Optical aberrations in a spatial telescope may be issued from mirrors manufacturing imperfection, misalignment of the optical combination mirrors, sensitivity to thermal gradients. They are fully described by the WFE function, which is the phase difference between a reference spherical wave front centered on the detector plane and the real wavefront. WFE is thus a 2D function defined over the telescope pupil.

Considering a circular pupil shape, WFE is usually expanded over a set of orthonormal circular functions, the Zernike polynomials $\{Z_i\}$.

Zernike polynomial expansion is interesting because one may associate a particular aberration to each polynomial. For instance, Z_4 corresponds to a pure defocus.

There is a mathematical link between WFE, the occultation mask P and the Optical Fourier Transform for a monochromatic light with wavelength λ : OTF is the autocorrelation of the complex pupil function P_c defined by (1)

$$P_c = P e^{\frac{j2\pi}{\lambda} WFE} \quad (1)$$

The Wiener Kinchine theorem allows to write

$$OTF = FT \left(\left| FT^{-1} \left(P e^{\frac{j2\pi}{\lambda} WFE} \right) \right|^2 \right)$$

The ideal case is a constant WFE, which means that OTF is the autocorrelation of the occultation binary mask.

A useful aberration figure consists in the standard deviation of WFE, σ_{WFE} , computed over the occultation mask. What matters is the ratio σ_{WFE}/λ that should be kept under 1/20 in order to consider the telescope as a diffraction limited one. For a classical panchromatic band centered on $\lambda=650\text{nm}$, this leads to $\sigma_{WFE}<32.5\text{nm}$.

Fig. 2 depicts an example of likely post Pleiades telescope WFE comprising some high order aberrations with $\sigma_{WFE}=63\text{nm}$, with the corresponding global MTF (including detector MTF).

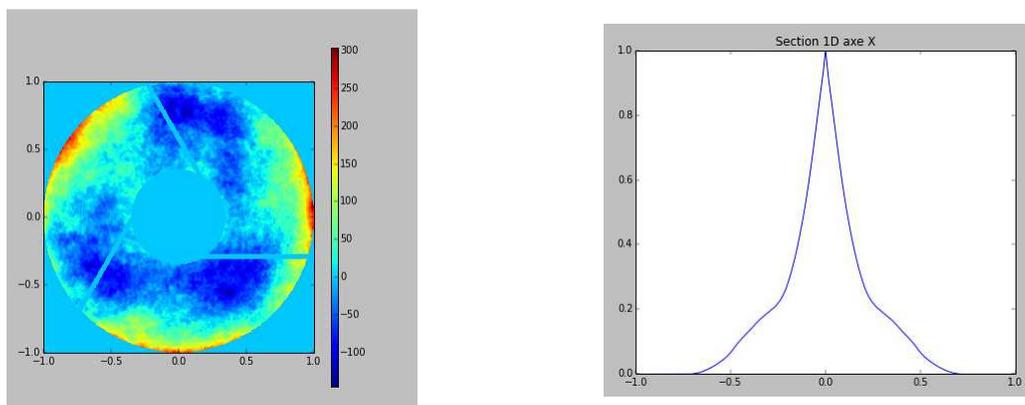


Fig 2 WFE map (left) and corresponding MTF(right)

C. Classical systems

Earth observing satellites have usually refocusing capabilities but nothing else. Refocusing capability is mandatory as a consequence of the launch, the gravity modification and some desorption effects in the spatial environment. For instance, Pleiades-HR satellites secondary mirror M2 may be translated using a thermal command [1, 2].

Defocus has to be measured and there are several ways of achieving such a task.

Usually, there is no onboard defocus assessment device and one has to rely on external known object, for instance stars if the satellite platform has a star pointing capability. It may be measured using ground knife edge targets, but cloud coverage may prove annoying.

On these classical systems, the refocusing operation is an open loop: defocus is assessed on ground and the corrective command, once computed, is uploaded to the satellite.

Refocusing tasks are then very rarely done, except during the inflight commissioning period because of desorption phenomenon. A typical routine refocusing time interval might be 6 months or one year.

Such systems are thus thought to be nearly very stable and with no default except defocus. This means of course very stringent constraints for large telescope realization.

Introducing active optics may improve significantly this situation.

D. Active optics: principles and advantages

Active optics consists in measuring the optical aberrations and correcting them with onboard devices, using a closed loop, meaning no ground operations needed. The assessment device is called a Wave Front Sensor (WFS) and there are several options for the correcting devices: secondary M2 mirror with enhanced steering capability, introduction of a steerable mirror the deformation of which compensates for the aberrations.

Such a concept allows to fit evolving aberration that could appear along the orbit due for instance to thermal environment and the correction is not limited to defocus but also covers higher order aberrations.

It helps to maximize the MTF and get closer to the diffraction upper bound even with compact optical combination or lighter primary mirror.

Active optics is well known from on ground astronomers because of atmospheric turbulence that limit the angular resolution to λ/r_0 , where r_0 is the Fried radius, typically equal to 10cm : it means that without active optics, it would be useless to build on ground telescope with diameter greater than 10cm.

However, the task is hopefully much easier for satellites because one has only to deal with static or slowly evolving defaults and correction frequency is a matter of Hz instead of kHz for atmospheric turbulence.

Dynamically and accurately assessing the WFE is thus a major task.

Two WFS devices were analysed by CNES : Shack –Hartman [3] and Phase diversity devices.

We focused hereunder on the phase diversity study.

III. PHASE DIVERSITY TECHNIQUE

A. Principle

Phase diversity is based upon a double and simultaneous acquisition of the same object, the second acquisition being intentionally defocused. The defocus differential is assumed to be known.

Phase diversity principle is depicted on Fig.3.

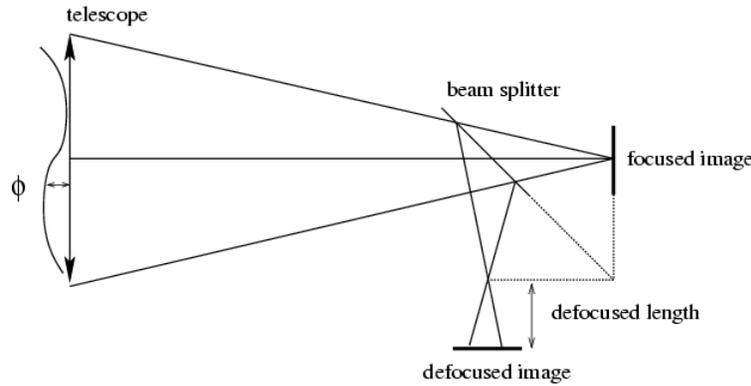


Fig. 3 Phase diversity principle

Phase diversity may be used with star as well as earth landscapes, the latter case being more complex to solve because the landscape is unknown.

From a theoretical point of view, the secondary defocused image is mandatory in order to get rid of the sign ambiguity of the symmetrical component of the WFE. From a more practical point of view, it allows to cancel the unknown landscape between the equations modelling each acquisition.

Active optics implies to solve the problem with earth landscape acquisitions, since aberrations may evolve during the viewing part of the orbit, due for instance to thermal environment sensitivity.

CNES has thus developed its own technique for WFE retrieval, which belongs to the “Inverse problem” mathematical category.

IV. CNES SOLVING ALGORITHM

The general idea is to model the two images formation using a classical acquisition model based upon convolution of the landscape with a Point Spread Function (PSF), instrumental noise and 2D sampling.

Taking the Fourier transform makes this model easier to handle since PSF convolutions become bare multiplicative product with the MTF corresponding to each acquisition.

$$\begin{aligned} FT(landscape) \times MTF_{detector} \times OTF(WFE) + FT(noise_1) &= FT(image_1) \\ FT(landscape) \times MTF_{detector} \times OTF(WFE + \Delta\alpha_4 Z_4) + FT(noise_2) &= FT(image_2) \end{aligned}$$

In these models, aliasing is considered as insignificant and WFE has to be computed. The differential defocus has been modelled as a pure Z_4 polynomial with known $\Delta\alpha_4$ weight.

Since only a limited number of aberrations are to be computed, WFE will be written as :

$$WFE = WFE_{a\ priori} + \sum_{i \in \mathfrak{S}} \alpha_i Z_i$$

where $\{\alpha_i\}_{i \in \mathfrak{S}}$ are the Zernike expansion coefficients corresponding to the aberrations to be searched. $WFE_{a\ priori}$ represents the a priori knowledge of the WFE, for instance very high order aberrations that have been measured on ground using interferometric measurements and assumed to be identical after launch. It could also represents the aberrations that have to be measured once after launch and that remains stable.

CNES algorithm is based upon the minimization of a functional $\mathcal{F}(WFE)$ issued from [4]

$$\begin{aligned} \mathcal{H}(v_x, v_y) &= \frac{FT(image_1) \times OTF(WFE + \Delta\alpha_4 Z_4) - FT(image_2) \times OTF(WFE)}{\sqrt{2\sigma_{bruit}^2 |OTF(WFE)|^2 + |OTF(WFE + \Delta\alpha_4 Z_4)|^2}} \\ \mathcal{F}(WFE) &= \sqrt{\iint_{v_x v_y} |\mathcal{H}(v_x, v_y)|^2 dv_x dv_y} = \sqrt{\iint_{x,y} |TF^{-1}(\mathcal{H})|^2 dx dy} \end{aligned}$$

Where σ_{bruit}^2 stands for the noise variance supposed to be identical for the two images.

The physical interpretation is quite straightforward: as long as WFE is significantly different from the true WFE, $TF^{-1}(\mathcal{H})$ is an image that contains structures linked to the underlying landscape that will finally disappear when finding the perfect solution WFE. $TF^{-1}(\mathcal{H})$ is then a pure noise image with unity variance.

Getting back to the spatial domain is interesting in order to get rid of border effects by limiting the spatial integration domain of $|TF^{-1}(\mathcal{H})|^2$.

When taking the initial Fourier transform of the two images, a classical mirror symmetry is applied in order to limit the artefacts due to implicit signal periodization.

Functional minimization relies on a classical Levenberg-Marquardt iterative algorithm.

This algorithm was tested through a benchmark with a 30cm resolution instrument.

V. BENCHMARK PRESENTATION

A. Instrumental hypothesis and simulations

High resolution systems usually comprise a high resolution panchromatic band with a large spectral bandwidth and several lower resolution multispectral bands. The idea is to use phase diversity images at the panchromatic resolution in order to have a better sensitivity to aberrations since Nyquist panchromatic frequency is four times larger than multispectral one and pretty close the cut-off frequency, as mentioned in paragraph II.A.

Phase diversity images are supposed to be delivered by a specific onboard device, according to the principle depicted in Fig. 3.

However, Signal to Noise ratios (SNR) for WFS device images may be significantly lower than for main instrument panchromatic images, for two reasons :

- The beamsplitter divides the light flux by 2
- WFS spectral bandwidth may be intentionally narrower in order to be closer to the implicit monochromatic assumption when using (1) for image formation model within the phase diversity algorithm.

SNR WFS assumptions are summed up in Tab. 1. L1 is representative of dark zones like shadows, L2/2 is a rather most likely radiance to be observed.

Tab1 WFS Signal to Noise ratios

	L1	L2/2	L2
Radiances ($\text{Wm}^{-2}\text{Str}^{-1}\mu\text{m}^{-1}$)	13	47.5	95
SNR	11	34	58

Phase diversity images, with size 128x128 pixels, were simulated from airborne 10cm images taking into account these SNR assumptions as well as $\text{MTF}_{\text{detector}}$, OTF and 12 bits quantization. Each airborne image yields 100 phase diversity image couples. Mean radiance for each set of 100 images issued from the same airborne image was fixed to a L2/2 target value.

B. Key parameters

Sensitivity of WFE assessment accuracy to several key algorithm parameters had to be assessed. CNES has undertaken a sensitivity analysis to the following criteria :

- WFE

Even for a given overall σ_{WFE} , WFE differs from each other according to the relative amount of high order aberrations as well as their nature (high order aberrations may look like white noise or contains structures due to mirror realization technique).

6 WFE were tested during the benchmark, that are depicted on Fig 4.

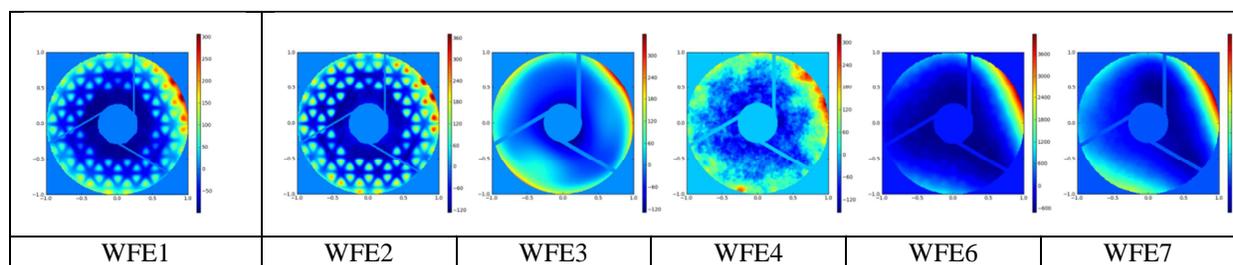


Fig 4 Benchmark Wave Front Error set

- Searched aberrations $\{\alpha_i\}_{i \in \mathcal{S}}$

Algorithm depends on $\{\alpha_i\}_{i \in \mathcal{S}}$ Zernike coefficients that are looked after. 3 Zernike sets were envisaged:

$\{\alpha_4\}$: this case corresponds to defocus estimation only

$\{\alpha_4, \alpha_5, \dots, \alpha_{13}\}$: 10 aberrations are estimated.

$\{\alpha_4, \alpha_5, \dots, \alpha_{36}\}$: 33 aberrations are estimated.

One may wonder why $\{\alpha_1, \alpha_2, \alpha_3\}$, respectively referring to Piston, Tip and Tilt aberrations, are not included the estimated set. The reason is simple : piston has no impact upon OTF and Tip/Tilt only induces spatial images translation but no differential effects between the two images. These aberrations thus cannot be assessed through phase diversity technique using earth images.

- $WFE_{\text{a priori}}$ knowledge

Better results are expected when using a priori knowledge. For each three cases of searched aberrations set, two options were tested : perfect knowledge of unsearched aberrations or complete lack of knowledge.

- Differential defocus

A very small value of differential defocus means the two images are very similar and the functional value $\mathcal{F}(WFE)$ will be close to unity, even if WFE assumption is significantly false. On another hand, large differential defocus values will wipe out the high frequency content of the secondary image. Three differential defocus values were tested : $\Delta\alpha_4=50\text{nm}$, $\Delta\alpha_4=190\text{nm}$ and $\Delta\alpha_4=300\text{nm}$

- Landscapes

Uniform landscapes (glassy lakes, clouds etc...) should be less suited to phase diversity estimation than textured scenes with high frequency content. Representative simulations of phase diversity image couples were computed from four airborne 10cm images with very different type of frequency content : Amiens, Cannes, La Crau and Marseille [4].

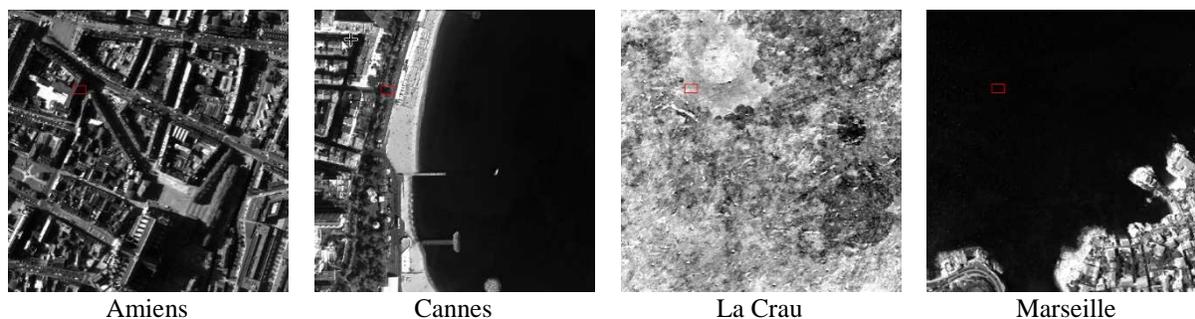


Fig 4 Airborne images for Phase Diversity benchmark

VI. MAJOR RESULTS

The benchmark delivers a rather huge quantity of data when combining the numerous variables :

- 400 128x128 pixels zones
- 6 WFE
- 3 sets of searched aberrations
- 2 assumptions for knowledge/ignorance of $WFE_{\text{a priori}}$
- 3 differential defocus values

This amounts to $400 \times 6 \times 3 \times 2 \times 3 = 43200$ algorithm runs, that allows to get a thorough analysis.

Here are the main conclusions.

A. *WFE*

WFE6 and WFE7 are very worst case WFE with standard deviation close to the wavelength λ . Not surprisingly, phase diversity algorithm does not manage to solve the problem except when only defocus is searched with the a priori knowledge of other aberrations. For more likely WFE, phase diversity happens to be dependent upon the nature of high order aberrations, when considered as unknown: WFE4 leads to slightly better results than WFE1 with nearly the same high order aberrations (above Z36) RMS value. The gap is much more significant with WFE2, which looks like WFE1 but with enhanced high order aberrations. A priori knowledge of high order aberrations is thus more important when these aberrations contains some discontinuities.

B. *Landscape type*

As expected, the landscape frequency content is of utmost importance. Amiens yields excellent results while Cannes results are clearly below but surprisingly, even zones containing very sparse structures yields good WFE estimation.

Quantitatively, for Amiens, with WFE4 and 190nm differential defocus, when looking just for Z4 and having knowledge all other aberrations, mean error is 0.45nm and max error is 2.57nm. On the whole image set, mean error value is 5.04nm and 86.25% of estimations are better than 5nm.

Tab 2 gives the percentage of results better than 20nm when searching Z4 only, Z4-Z13 or Z4-Z36 aberrations with WFE4 and 190nm differential defocus, with an a priori knowledge of other aberrations

C. *Searched aberrations*

Accuracy was expected to go down when increasing the number of searched aberrations and the experiment confirmed this forecast, as shown in Tab 3.

D. *WFE a priori knowledge*

The a priori knowledge of not searched aberrations is an asset that proves not significant when the unknown part of the WFE contains no structure (for instance WFE4) but very significant for WFE2 which contains a significant amount of high order aberrations with very structured content. As depicted on Tab 4, phase diversity algorithm fails except for Z4-limited assessment when ignoring the other aberrations.

E. *Differential defocus*

When estimating Z4 only, the three tested values of differential defocus, respectively 50nm, 190nm and 300nm, roughly lead to the same accuracy. However, when assessing more high order aberrations, 50nm value proves to depart from the two other options, as depicted by Tab 5 corresponding to Z4-Z36 estimation.

Tab 2 : Sensitivity to landscapes

Landscape	AMIENS			CANNES			LA_CRAU			MARSEILLE		
	4	4-13	4-36	4	4-13	4-36	4	4-13	4-36	4	4-13	4-36
Searched aberrations	4	4-13	4-36	4	4-13	4-36	4	4-13	4-36	4	4-13	4-36
% of results better than 20nm	100	100	94	69	46	39	100	99	1	100	98	60

Tab 3 Sensitivity to searched aberrations number

WFE	1	1	1	2	2	2	3	3	3	4	4	4
Searched aberrations	4	4-13	4-36	4	4-13	4-36	4	4-13	4-36	4	4-13	4-36
% of results better than 20nm	92	86	48	92	81	45	94	85	46	92	85	48

Tab 4 . Sensitivity to WFE a priori knowledge.
Percentage of estimation better than 20nm with differential defocus=190nm

WFE a priori knowledge : True	WFE1	WFE2	WFE3	WFE4
Z4 seul	93	92	95	92
Z4-Z13	86	81	86	86
Z4-Z36	48	46	46	48
WFE a priori knowledge : False				
Z4 seul	94	85	92	92
Z4-Z13	74	0	76	85
Z4-Z36	38	0	46	42

Tab 5 Sensitivity to differential defocus for Z4-Z36 estimation

WFE	1	1	1	2	2	2	3	3	3	4	4	4
Defocus	50	190	300	50	190	300	50	190	300	50	190	300
% better than 20nm	12	48	52	12	45	48	6	46	48	10	48	53

VII. CONCLUSION

Active optics is envisaged by CNES for its future very high resolution earth observation systems, as a mean to keep the telescope close to the diffraction limit while allowing evolving aberrations. Such a concept implies an onboard WFS, that could be based upon the Phase Diversity principle. In order to assess the accuracy of the phase diversity technique for WFE measurement, CNES has developed its own phase diversity algorithm and undertaken a thorough sensitivity analysis of its performances, based upon representative simulations of phase diversity image couples. This study involved main parameters such as landscapes, differential defocus value, WFEs, number of searched aberrations or WFE a priori knowledge.

The overall conclusion is that phase diversity is an efficient and very accurate tool for WFE measurement for standard WFE. A 20nm target is easily obtained. Improvement now concentrates on algorithm simplification in order to makes easier its on board implementation.

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