Optical vortex behavior in dynamic speckle fields

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Abstract. The dynamic behavior of phase singularities, or optical vortices, in the pseudo-phase representation of dynamic speckle patterns is investigated. Sequences of band-limited, dynamic speckle patterns with predetermined Gaussian decorrelation behavior were generated, and the pseudo-phase realizations of the individual speckle patterns were calculated via a two-dimensional Hilbert transform algorithm. Singular points in the pseudo-phase representation are identified by calculating the local topological charge as determined by convolution of the pseudo-phase representations with a series of 2×2 nabla filters. The spatial locations of the phase singularities are tracked over all frames of the speckle sequences, and recorded in three-dimensional space (x, y, f), where f is frame number in the sequence. The behavior of the phase singularities traces 'vortex trails' which are representative of the speckle dynamics. Slowly decorrelating speckle patterns results in long, relatively straight vortex trails, while rapidly decorrelating speckle patterns results in tortuous, relatively short vortex trails. Optical vortex analysis such as described herein can be used as a descriptor of biological activity, flow, and motion. © *2012 Society of Photo-Optical Instrumentation Engineers (SPIE).* [DOI: 10.1117/1.JBO.17.5.050504]

Speckle fields arising from scattering media such as biological tissue contain locations of zero intensity and undefined phase. The phase in the immediate vicinity of these singular points rotates through a full 2π radians along a circular path surrounding these points. This spiral behavior of the phase values has lead to these singular points being referred to as optical vortices. Optical vortices may be described in terms of topological charge n_t with each vortex typically possessing a topological charge of ± 1, depending upon the direction of rotation of the phase values;[1,2] however, higher order charges are possible. As a direct outcome of the principle of conservation of charge, vortices only appear in pairs, with one member of the pair possessing a topological charge of $+1$ and the other pair member possessing a topological charge of -1. This further implies that optical vortices can only be created and destroyed in pairs. As the scattering particles in the dynamic media move, the phase in the scattered field, and therefore the locations of the optical vortices, also change in a related fashion. The behavior of optical vortices and beams that contain optical vortices has received much attention lately.[2–4] Much of this attention has focused on the density and behavior of the optical vortices as the field propagates through a turbulent atmosphere.[2,4] Sendra et al.[5] evaluated vortex behavior in dynamic speckle images, and pointed to the biological and industrial applications of analyzing vortex activity. They note there is a strong relationship between the motion of a scattering diffuser and the resultant motion of the optical vortices in the observed dynamic speckle pattern.

The apparent robustness of optical vortices combined with the observation that vortices in a dynamic, stochastic optical field move in a manner that correlates with the motion of scattering particles (or medium) implies that optical vortex behavior may be used as a surrogate for understanding the dynamic behavior of scattering media. Particular applications in biomedicine, where vortex analysis may be of use, include investigating the dynamic behavior of scattering suspensions, particle sizing, monitoring cellular activity, and microcirculation studies, for example. The purpose of this letter is to demonstrate one approach to studying optical vortex behavior in simulated speckle intensity patterns that exhibit dynamic behavior similar to that observed when coherent light is scattered by biological tissues.

To investigate the spatio-temporal behavior of optical vortices in dynamic speckle patterns, we generated sequences of band-limited dynamic speckle patterns in the Matlab environment using the numerical method we described in an earlier publication.[6] Using this approach, we were able to generate fully developed, dynamic speckle patterns with very well-controlled first and second order statistics, and also with prescribed decorrelation behavior. Following the generation of the speckle sequences, the pseudo-phase $\hat{\phi}$[7] of the speckle patterns was generated using a two-dimensional (2-D) Hilbert transformation of the speckle patterns. This representation of $\hat{\phi}$ is not unique, and an entirely different representation of the pseudo-phase can be generated if, for example, one were to employ a 2-D Fourier transformation in place of the Hilbert transformation. This is the classic phase-retrieval problem. Regardless of the numerical method used to generate the pseudo-phase, the operation simply exploits information already present in the signal without adding any new information.[5] Once a 2-D representation of $\hat{\phi}$ is generated, the singular points which identify the optical vortices can be located. These points then can be tracked as the speckle pattern dynamically evolves.

We begin our formal discussion of the problem by noting that the complex representation of an optical field and its relationship to the intensity image is given by

$$U(x, y, z; t) = A(x, y, z) \exp[i\omega t]$$

$$I(x, y, z) = \lim_{T \to \infty} \int_{-T/2}^{T/2} |U(x, y, z; t)|^2 dt \qquad (1)$$

$$= |A(x, y, z)|^2,$$

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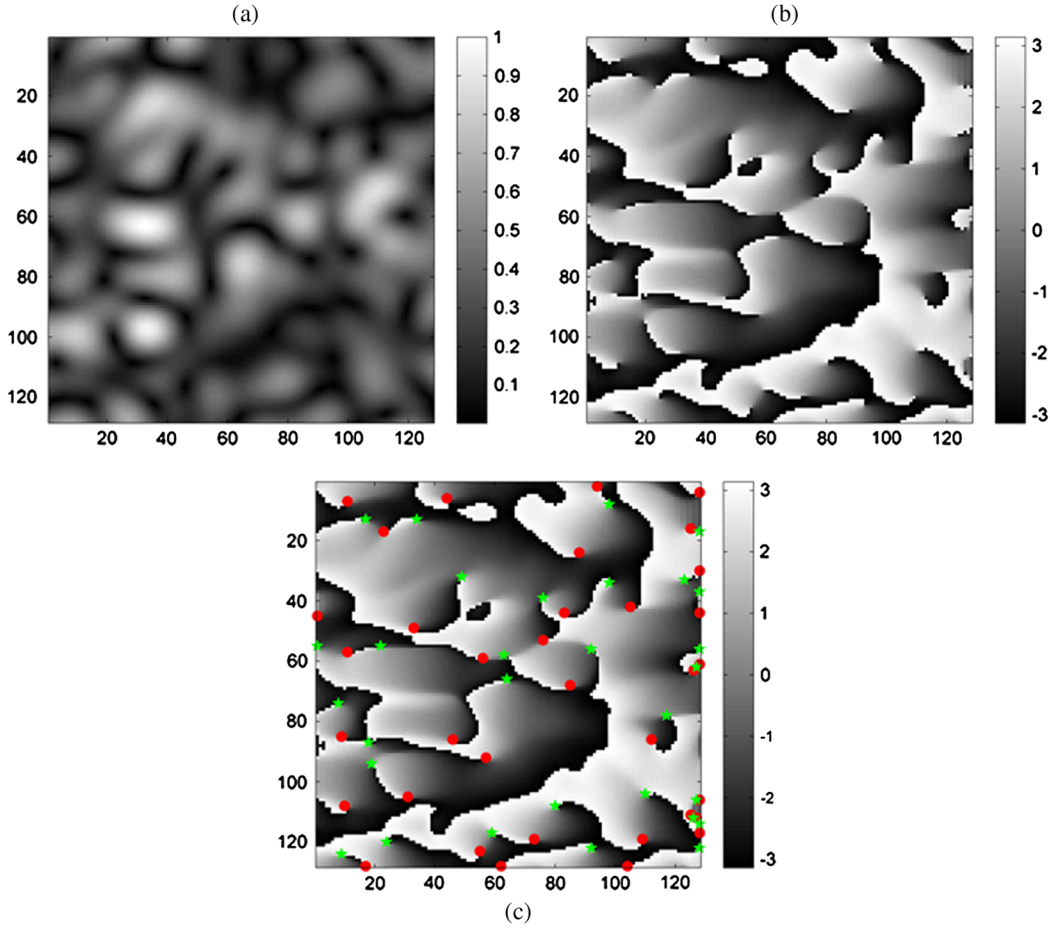
(a)

(b)

(c)

Fig. 1 Example of a band-limited numerically generated speckle pattern (a), its pseudo-phase representation (b), and the pseudo-phase representation with the positively charged (green stars) and negatively charged (red circles) optical vortices identified (c).

where the phasor amplitude of the field, A, is itself a complex-valued function of space. The local true phase of the field is defined as usual as $\phi(x,y) = \tan^{-1} \text{Im}[U(x,y)]/\text{Re}[U(x,y)]$. To generate a 2-D representation of the pseudo-phase $\hat{\phi}(x,y)$ from $I(x,y)$ we use a Hilbert transform in two dimensions, such that
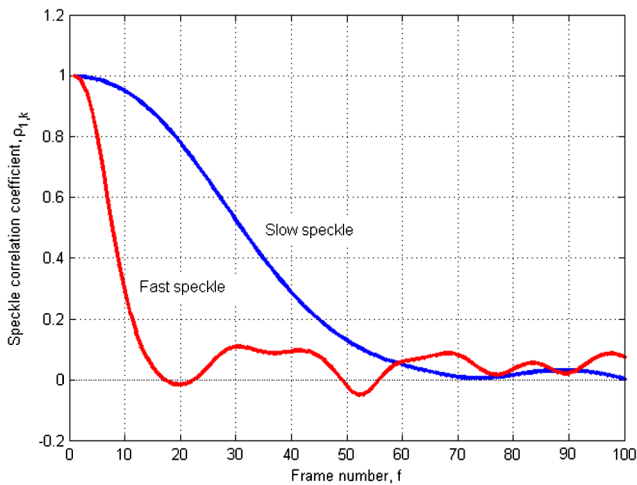

Fig. 2 Autocorrelation functions of the rapidly decorrelating speckle sequence versus the more slowly decorrelating speckle sequence used in this study.

$$\hat{U}(x,y) = I(x,y) + jH\{I(x,y)\} \quad (2)$$

and

$$\hat{\phi}(x,y) = \tan^{-1}\frac{\text{Im}\{\hat{U}(x,y)\}}{\text{Re}\{\hat{U}(x,y)\}}.$$

The phase singularities are defined in terms of n_t:

$$n_t \equiv \frac{1}{2\pi} \oint_c \nabla\hat{\phi}(x,y) \cdot d\vec{l}, \quad (3)$$

where $\nabla\hat{\phi}(x,y)$ is the local phase gradient, and the contour integral is taken over path l on a closed loop c around the vortex. It is clear that $n_t = 0$ everywhere $\hat{\phi}$ is differentiable, except at the singularity where the phase is undefined. Since $\hat{\phi}$ is a continuous function, and has continuous first derivatives, by Stokes theorem we can see that

$$n_t \equiv \frac{1}{2\pi} \oint_c \nabla\hat{\phi}(x,y) \cdot d\vec{l} = \frac{1}{2\pi} \iint_D \left(\frac{\partial^2\hat{\phi}}{\partial x \partial y} - \frac{\partial^2\hat{\phi}}{\partial y \partial x}\right) \partial x \partial y, \quad (4)$$

where D is a disc of radius a enclosing the circular path over which n_t is evaluated.[8,9] This formulation suggests the locations of the optical vortices can be determined efficiently through a series of convolution operations:
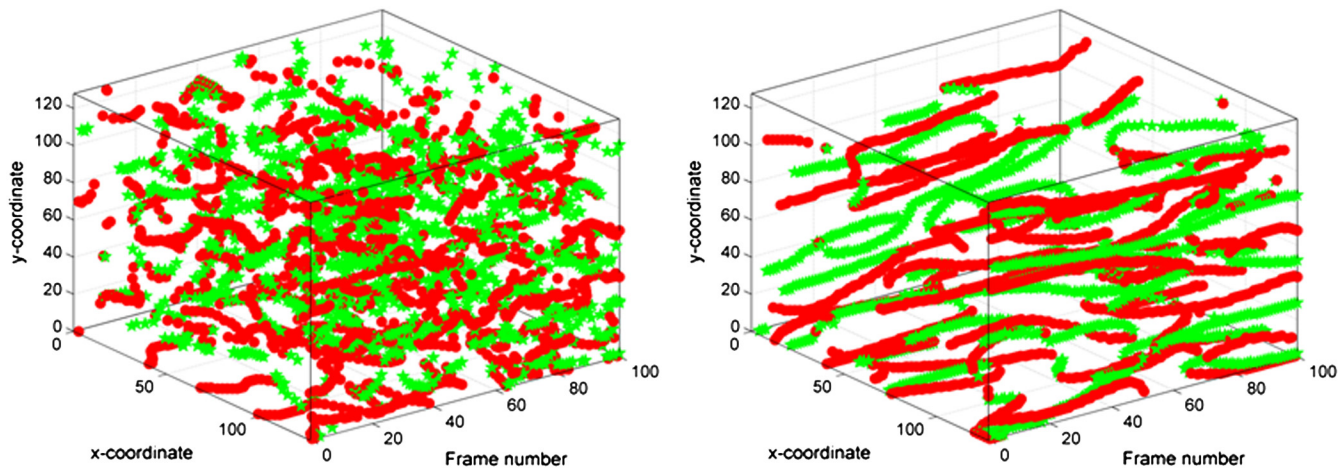
Fig. 3 Three-dimensional representation of the vortex trails for the rapidly decorrelating speckle sequence (a), and the more slowly decorrelating speckle sequence (b). The green stars represent the positively charged vortices and the red circles represent the negatively charged vortices.

$$n_t = \hat{\phi}(x,y) \otimes \nabla_1 + \hat{\phi}(x,y) \otimes \nabla_2 + \hat{\phi}(x,y) \otimes \nabla_3 + \hat{\phi}(x,y) \otimes \nabla_4,$$

where

$$\nabla_1 = \begin{pmatrix} 0 & 1 \\ 0 & -1 \end{pmatrix}; \quad \nabla_2 = \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix};$$

$$\nabla_3 = \begin{pmatrix} -1 & 0 \\ 1 & 0 \end{pmatrix}; \quad \nabla_4 = \begin{pmatrix} 0 & 0 \\ -1 & 1 \end{pmatrix};$$

(5)

and $\otimes$ is the convolution operator. Figure 1(a)–1(c) displays a numerically synthesized speckle pattern, its pseudo-phase representation, and the pseudo-phase representation with the optical vortices indicated by the superimposed stars and circles, respectively. Here, the vortices with positive topological charge are indicated by green stars and those with a negative topological charge by red circles.

Two sequences of 100 dynamic speckle patterns each were generated numerically. These speckle sequences displayed temporal decorrelation behavior, as shown in Fig. 2. From this figure, we see that both sequences exhibited Gaussian decorrelation behavior, but with drastically different characteristic times. The 'fast' sequence was defined by a time constant of $\tau_f \cong 9$ frames, while the 'slow' sequence was characterized by a longer time constant $\tau_s \cong 36$ frames. For both sequences, pseudo-phase representations were generated for each frame, and the locations of the vortices identified. These locations were then plotted in (x, y, f), where f is frame number in the sequence. Following the convention of Fig. 1(c), the positively charged vortices were plotted in green stars, and the negatively charge vortices were plotted in red circles.

The results of this operation are shown in Fig. 3(a) and 3(b). Several observations can be made regarding these figures. A key observation is the vortices are robust. That is, individual vortices persist through at least several frames, even in the rapidly decorrelating speckle example. The locations of the individual vortices over several frames trace a path, or a vortex trail. In the rapidly decorrelating speckle sequence, the vortex trails are relatively short and tortuous [Fig. 3(a)]; however, in the slowly varying sequence, the vortex trails are quite long, and trace a much straighter path [Fig. 3(b)]. It is clear that in the limit of a truly static speckle sequence, the vortices would maintain their (x, y)

positions through all frames f. Careful examination of Fig. 3(a) and 3(b) also reveals locations where positively and negatively charged optical vortices annihilate each other. Elsewhere, new vortices are formed. These events always occur in pairs (i.e., both a positively and a negatively charged vortex is either destroyed or created), and arise because of the conservation of charge principle.[4]

In an actual, physical setting, the motion of the speckles arising from scatter from biological tissues arises from numerous sources, including the random motion of cellular activity, and from the more-ordered motion of red blood cells, for example. We may generalize and simply say that motion in these speckle patterns is from biological activity. From this discussion, it becomes apparent that the spatio-temporal behavior of optical vortex trails is an indicator of biological activity, and specific imaging configurations and experiments could be designed to focus on one particular type of motion. For example, one could coherently image cell sheets, and use the above vortex analysis to quantify cellular metabolic activity. Alternatively, it is feasible that one could coherently image the microcirculation, and use vortex analysis to study microcirculatory flow.

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